

Basins of distinct asymptotic states in the cyclically competing mobile five species game

Beomseok Kim, and Junpyo Park

Citation: *Chaos* **27**, 103117 (2017); doi: 10.1063/1.4998984

View online: <https://doi.org/10.1063/1.4998984>

View Table of Contents: <http://aip.scitation.org/toc/cha/27/10>

Published by the [American Institute of Physics](#)

Articles you may be interested in

[Attractors in complex networks](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **27**, 103105 (2017); 10.1063/1.4996883

[Opinion diversity and community formation in adaptive networks](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **27**, 103115 (2017); 10.1063/1.4989668

[Epidemic spreading between two coupled subpopulations with inner structures](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **27**, 103104 (2017); 10.1063/1.4990592

[Classification of complex networks based on similarity of topological network features](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **27**, 091102 (2017); 10.1063/1.4997921

[Transports in a rough ratchet induced by Lévy noises](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **27**, 103102 (2017); 10.1063/1.4996264

[Observability and synchronization of neuron models](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **27**, 103103 (2017); 10.1063/1.4985291



Basins of distinct asymptotic states in the cyclically competing mobile five species game

Beomseok Kim¹ and Junpyo Park^{2,a)}

¹*Department of Mathematics, KNU-Center for Nonlinear Dynamics, Kyungpook National University, Daegu 41566, South Korea*

²*Department of Mathematical Sciences, Ulsan National Institute of Science and Technology, Ulsan 44919, South Korea*

(Received 3 August 2017; accepted 2 October 2017; published online 13 October 2017)

We study the dynamics of cyclic competing mobile five species on spatially extended systems originated from asymmetric initial populations and investigate the basins for the three possible asymptotic states, coexistence of all species, existences of only two independent species, and the extinction. Through extensive numerical simulations, we find a prosperous dependence on initial conditions for species biodiversity. In particular, for fixed given equal densities of two relevant species, we find that only five basins for the existence of two independent species exist and they are spirally entangled for high mobility. A basin of coexistence is outbreaking when the mobility parameter is decreased through a critical value and surrounded by the other five basins. For fixed given equal densities of two independent species, however, we find that basin structures are not spirally entangled. Further, final states of two independent species are totally different. For all possible considerations, the extinction state is not witnessed which is verified by the survival probability. To provide the validity of basin structures from lattice simulations, we analyze the system in mean-field manners. Consequently, results on macroscopic levels are matched to direct lattice simulations for high mobility regimes. These findings provide a good insight into the fundamental issue of the biodiversity among many species than previous cases. *Published by AIP Publishing.* <https://doi.org/10.1063/1.4998984>

Species coexistence is a main issue in ecological sciences. In nature, competitions among species can be elucidated by predator-prey systems, and evolutionary games are powerful tools to interpret dynamical phenomena. In the past several years, one of the popular children's games, the rock-paper-scissors (RPS) game, has been widely used to describe cyclic competing structures among species in nature. As an extension, the generally extended rock-paper-scissors (ERPS) game has been adopted to describe the cyclic competition for many species. As above, existing studies for extended rock-paper-scissors games have been investigated about the influence of external interactions, for instance an individual's mobility, to promote the biodiversity with a symmetric single initial density on spatially extended systems in frameworks on the microscopic level because of the dilemma of unstable coexistence of all species at the macroscopic level. One of the existing studies for cyclic competing three species addressed the issue of asymmetric initial densities and suggested the global insight for species' coexistence/extinction related to initial densities of all species on spatially extended systems. Motivated from these two studies, we consider the relation between asymmetric initial densities and biodiversity on the cyclic competing five species model which possesses three different survival states. An impediment for suitable representations of the phase space to illustrate the underlying five dimensional system, we compute basin structures

of initial values on a three dimensional triangular phase space, which is similar to the simplex representation, satisfying the sum of three components as certain values with fixing initial values for the other two components. We characterize the final states to demonstrate basin structures for possible survival states from direct simulations of the microscopic models. Through our extensive numerical simulations on a lattice, we find that symmetry-breaking of initial densities can lead only two different survival states, the existence of two species, and the coexistence of all species. On the contrary, to rock-paper-scissors models, the extinction state that only one species exists does not appear and this fact can be guaranteed by the measure of the survival probability. Especially, basin structures on a triangular phase space are totally different depending on the consideration that which initial densities of two species are fixed. For fixed two neighbor species, the basin for the coexistence of all species can emerge at the concurrence of spirally entangled five basins for the existence of two species. For fixed two independent species, however, the system yields different results. In this case, basins for the existence of two species are not spirally formed and only some cases of two species' existences can appear depending on the initial considerations. Further, for high mobility cases, these dynamical phenomena can be supported by mean-field frameworks. Our results will provide a qualitative insight into the biodiversity of more species systems.

^{a)}Electronic mail: jppark@unist.ac.kr

I. INTRODUCTION

Understanding mechanisms of species diversity is a fundamental problem in interdisciplinary science. Key factors to biodiversity is the survival of species which have been studied theoretically and experimentally.^{1–5} In this respect, various models and experiments suggested that nonhierarchical, cyclic competition among species is a significant framework for species coexistence which can be described by one of the evolutionary games, the rock-paper-scissors (RPS) game.⁶ The RPS nature has been a powerful tool to demonstrate and interpret various dynamical phenomena in ecosystems, for instance, matings of side-blotched lizards in California,⁷ colicinogenic microbes' competitions,⁸ rivalry among strains of yeast,⁹ and *Escherichia Coli*.¹⁰

Studies of the RPS game have been widely performed by incorporating spatial structures to be consistent with experimental observations due to the structurally unstable coexistence on macroscopic frameworks. In this regard, the coexistence of three species can occur by formation of entangled spiral waves in the spatial region and the individual's mobility has been identified as the key role for the coexistence/extinction.^{11,12} Subsequently, spatially extended systems provide various perspectives of key factors on species coexistence such as the effect of community and population size,^{13–15} coevolution in a well-mixed population,¹⁶ long-range migration,^{17,18} role of inhomogeneous or random reaction rates,^{19,20} local habitat suitability,²¹ and intraspecific competition.²²

Even much work has been performed on spatially extended systems, most of the existing studies have been considered for the RPS game. Although it is found that simply extending the numbers of species on cyclic competing structures may cause more involved phenomena.^{23–27} Accordingly, the dynamics for cyclic competing five species models such as the extended rock-paper-scissors (ERPS) or rock-paper-scissors-lizard-spock (RPSLS) games among five species on spatially extended systems have been investigated.^{28–35}

In existing studies on spatially extended systems, dynamical phenomena of species have been considered originated from the single initial condition. Actually, the dependence on initial conditions for a system's behavior is an important issue in dynamical systems: positive topological entropy in dynamical systems,³⁶ characterization of the space time complexity of cellular automata,³⁷ collective sensitivity between the original and induced hyper systems,³⁸ the emergence of strange attractors,³⁹ control of dynamical systems depending on initial conditions,^{40–42} basin stability for bistable states in complex systems,⁴³ and biodiversity depending on initial conditions in multispecies systems.^{44–47} Especially, in Refs. 46 and 47, the effect of initial conditions of the RPS game on spatially extended systems has been investigated. Depending on the choice of the mobility value, the available phase space is divided either into three regions for the extinction or into four regions including the basin of the coexistence in a proper representation. Consequently, for two distinct asymptotic states, extinction and coexistence, the rich and sensitive dependence of dynamical properties on initial conditions has been covered *versus* mobility. For the ERPS game, computations and analysis of all previous studies have been carried

out with respect to a single initial consideration with equal species densities. The aim of this paper is to provide more comprehensive insights into the issue of basins in the spatial ERPS game.

In Sec. II, we describe the spatial ERPS game of mobile five species and the method for computing basins for three distinct categories of asymptotic states. In Sec. III, we characterize the basins of distinct final states from lattice simulations. We consider that densities of the two species will be fixed under two different choices to be represented properly. To guarantee the appearance of final states, we provide the evidence by exploring the survival probability. In Sec. IV, we elucidate the basin structures at high mobility values theoretically based on ordinary differential equations (ODEs). Conclusion and further discussions will be addressed in Sec. V.

II. CYCLIC COMPETITION AMONG FIVE SPECIES ON SPATIALLY EXTENDED SYSTEMS

The cyclic competition model among mobile five species was proposed in Ref. 33 on a square lattice of size $N = L \times L$ with periodic boundary conditions. Each site of a lattice is either occupied by randomly distributed five species or left empty. An individual on a lattice site interacts with one of the four nearest neighbors, according to the following rules:

$$\begin{aligned} AB &\xrightarrow{\sigma} A\emptyset, & BC &\xrightarrow{\sigma} B\emptyset, & CD &\xrightarrow{\sigma} C\emptyset, \\ DE &\xrightarrow{\sigma} D\emptyset, & EA &\xrightarrow{\sigma} E\emptyset, \end{aligned} \quad (1)$$

$$\begin{aligned} A\emptyset &\xrightarrow{\mu} AA, & B\emptyset &\xrightarrow{\mu} BB, & C\emptyset &\xrightarrow{\mu} CC, \\ D\emptyset &\xrightarrow{\mu} DD, & E\emptyset &\xrightarrow{\mu} EE, \end{aligned} \quad (2)$$

$$XY \xrightarrow{\varepsilon} YX, \quad \forall X, Y \in \{A, B, C, D, E, \emptyset\}, \quad (3)$$

where A, B, C, D , and E represent five species and \emptyset denotes an empty site. Relations (1) indicate the interspecific competition between two different species with a rate σ . Relations (2) describe the reproduction of each species with a rate μ and it occurs when an empty space in neighbors is allowed. The exchange process (3) can occur for all pairwise individuals with the same rate ε defined by $\varepsilon = 2MN$ for an individual's mobility M and a system size N according to the theory of random walk.⁵⁰ The rates of interspecific competition, reproduction, and exchange are normalized as $\sigma/(\sigma + \mu + \varepsilon)$, $\mu/(\sigma + \mu + \varepsilon)$, and $\varepsilon/(\sigma + \mu + \varepsilon)$, respectively. In the simulation, an actual time step is defined when, on average, each individual has interacted with others once, i.e., one time step involves N pairwise interactions altogether.

In Refs. 46 and 47, the concept of attraction basin in the RPS game has been introduced. In these models, since the initial densities of three species are defined in a triangular region, the simplex S_3 can be a proper phase space to be represented. In the ERPS game, however, the initial densities of five species are originally defined by the simplex S_5 , so it is hard to demonstrate visually. To overcome this problem, by imitating the methods,^{46,47} we fix the initial densities of two species and thus all possible combinations of the initial

densities of remaining three species can be defined in a triangular phase space. In the triangular phase space, the coordinates of a point indicate a combination of the initial densities of three species and the total densities of five species will satisfy the simplex S_5 by incorporating the assumption of densities of two species. We will introduce the concrete method in Sec. III. We already know that the ERPS game possesses three distinct asymptotic states.³³ From the measure of the survival probability, we find that only two types of final states can appear in the ERPS game. The basins can thus be represented by regions in the triangular phase space with six different colors.

In Ref. 33, the interplay between mobility and coexistence has been investigated, but for the symmetrical case when the initial populations of the five competing species are identical. For $M < M_c$, coexistence of five species arises and the biodiversity is broken for $M > M_c$, where a critical value of mobility $M_c = (3.5 \pm 0.5) \times 10^{-4}$. To make an unbiased comparison with the previous work, we assume equal reaction probabilities for interspecific competition and reproduction rates, i.e., $\sigma = \mu = 1$ to be handled the mobility M as the only parameter of the system. To search for a plausible foundation, we use 30 random realizations on square lattices of 100×100 sites and the simulation time $T = 10N$ to ensure that the final state can be reached from any initial densities in our simulations.

III. BASIN STRUCTURE FROM LATTICE SIMULATIONS

To investigate the dependence on initial densities, we examine the basins of three distinct categories of asymptotic states: (a) \mathbf{p}_1 for the extinction, (b) \mathbf{p}_2 for the existence of two independent species which have no predator-prey relationship,

and (c) \mathbf{p}_5 for the coexistence of all species, which are predicted from the mean-field approach.³³ The phase space of the system can be defined by the five population densities, n_a, n_b, n_c, n_d , and n_e , and the ratios of the populations to the total number of lattice sites for the species A, B, C, D , and E , respectively.

For the sake of convenience for graphical representation of basins, we consider a triangular phase space $n_x + n_y + n_z = \alpha$ which stands for the fixation of the sum for initial densities α of arbitrary three species. From various constructions of initial densities, we consider that densities of remaining two species will be equally fixed by $(1 - \alpha)/2$ in a simple way to include the case of symmetric initial condition. On the construction of initial conditions, we consider two cases of initial constructions of three species based on cyclic predation ways: (a) within two neighboring pairs and (b) one neighboring pair with one independent species. To be addressed, sampled initial states will be considered as Δ_{ABC} and Δ_{ACE} for (a) and (b), respectively.

A. Initial conditions with equally fixed two neighboring species

For fixed equal densities of two neighboring species D and E , i.e., $n_d = n_e$, we demonstrate the basin structures of survival states on a phase space $n_a + n_b + n_c = \alpha$ for different values of the mobility M as shown in Fig. 1. The sum α of densities of three species A, B , and C is fixed to be 0.4, 0.6, and 0.8 from top to bottom, respectively. Hence densities of remaining two species D and E are equally fixed to be $n_d = n_e = 0.3, 0.2$, and 0.1 .

For $M > M_c$ as shown in Figs. 1(a), 1(f), 1(k) and 1(b), 1(g), 1(l) for $M = 3 \times 10^{-3}$ and 10^{-3} , respectively,

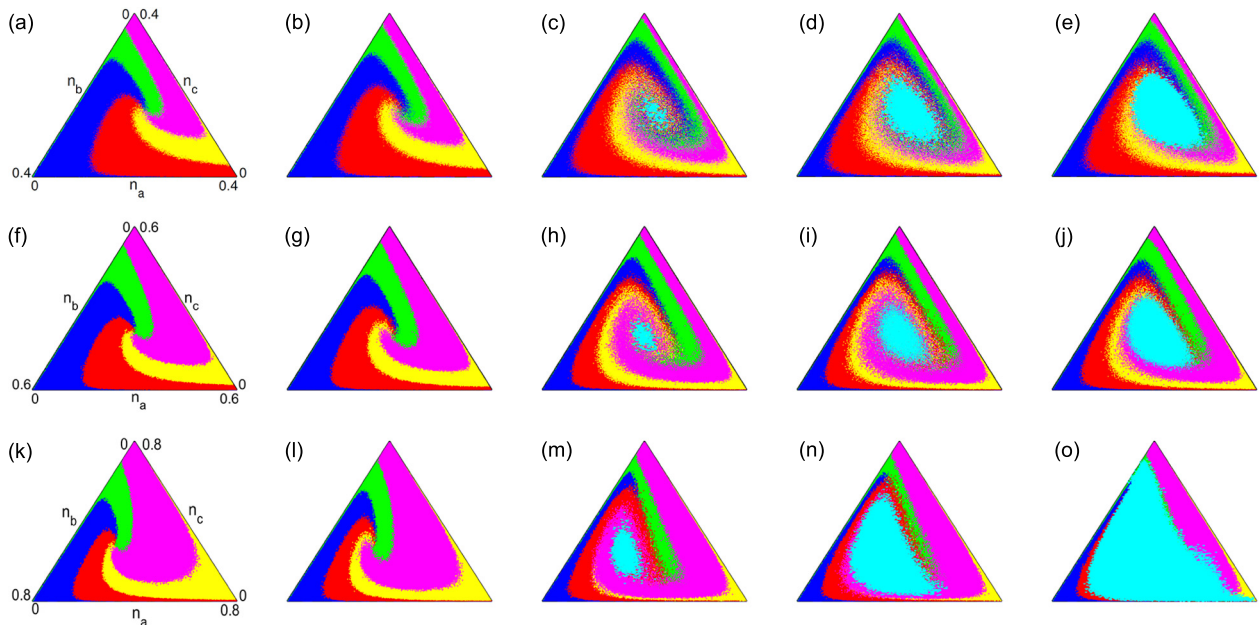


FIG. 1. Basins of final states of the ERPS game represented on the phase space Δ_{ABC} for different values of the mobility M from stochastic simulations. The coordinates denote the initial densities of the three species A, B , and C satisfying $n_a(0) + n_b(0) + n_c(0) = 0.4$ for (a)–(e), 0.6 for (f)–(j), and 0.8 for (k)–(o). For $M = 3 \times 10^{-3}$, in panels (a), (f), and (k), the phase space contains only five asymmetrically spirally entangled basins for \mathbf{p}_2 without that for the coexistence. These five basins are enlarged spirally to the boundaries of a phase space for $M = 10^{-3}$ as shown in panels (b), (g), and (l). For $M = 10^{-4}$, in panels (c), (h), and (m), a small basin for \mathbf{p}_5 with light blue color appears about the concurrence of five basins for \mathbf{p}_2 on Δ_{ABC} . For fourth and last columns, $M = 10^{-5}$ and 10^{-6} , respectively, the coexistence basin is more enlarged. For all possible considerations, the basin for \mathbf{p}_1 does not emerge. Each color represents the final state obtained from 30 random realizations on a given 100×100 square lattice.

regardless of the choice of the initial population densities, one of \mathbf{p}_2 which two independent species can survive is reached and the phase space is divided into five basins. Each colored region consisting of initial conditions that lead to one of \mathbf{p}_2 indicates: red for A and C , blue for A and D , green for B and D , magenta for B and E , and yellow for C and E , which are illustrated in Fig. 2.

Five basins of \mathbf{p}_2 are entangled asymmetrically and spirally and the concurrence of these basins is not necessary in the center of a triangular phase space of initial conditions. For the mobility $M = 3 \times 10^{-3}$, the concurrence among these basins is shifted as α is increased, which is illustrated in Figs. 1(a), 1(f), and 1(k). Along the boundaries between the basins, the final state of \mathbf{p}_2 is sensitive to small perturbations in the initial population densities. At the concurrence that five basins meet, an arbitrarily small variation can lead to a completely different state of \mathbf{p}_2 . Which two independent species can survive finally hence depends sensitively on small variations in the initial population densities.

As M is decreased through M_c , i.e., $M < M_c$, the \mathbf{p}_5 basin for the coexistence of five species emerges from the concurrence of all \mathbf{p}_2 basins as shown in Figs. 1(c), 1(h), and 1(m) for $M = 10^{-4}$. As illustrated in Figs. 1(d), 1(i), and 1(n) for $M = 10^{-5}$, the area of the \mathbf{p}_5 basin expands as M is decreased further, accompanied by the simultaneous shrinking of the five basins for \mathbf{p}_2 . Biodiversity is promoted by inhibiting population mobility, reflected by the augmentation of the area of the coexistence state \mathbf{p}_5 as shown in Figs. 1(e), 1(j), and 1(o) for $M = 10^{-6}$.

From basin structures, we may know that there is no basin of \mathbf{p}_1 for all values of mobility M . To be concrete whether \mathbf{p}_1 emerges or not, we investigate how three distinct states will occur frequently by calculating the survival probability P_{surv}^i of three distinct states for all values of mobility. Survival probability is defined by the probability of survival chances of each state for the given mobility value after sufficiently long enough simulation time and the indicator i is the number of existing species at the final step: $P_{surv}^i = \text{Prob}\{i\text{-surviving species after time } T\}$.

Without loss of generality, we measure the survival probability using the symmetric initial condition to be compared with the existing work.³³ In our simulations, we consider a simulation time $T = 4N$ to reach the steady state for

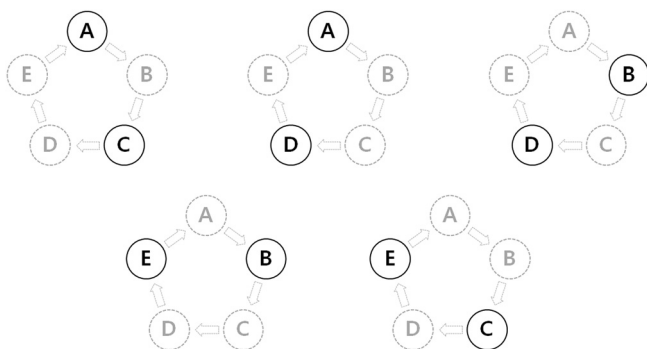


FIG. 2. Illustrations of five cases of \mathbf{p}_2 which indicates the existence of two species in the ERPS game. Interestingly, two species exist independent of interspecific competitions under the breaking of the cyclic structure.

100 random realizations at each mobility value with different lattice sizes.

Figure 3 shows the survival probability of three distinct states calculated for all values of mobility on different system sizes from 40×40 to 500×500 . For Fig. 3(a), the survival probability P_{surv}^1 for the state \mathbf{p}_1 is always zero. In other words, the extinction state has never occurred in the ERPS system even though it can be expected theoretically. For Fig. 3(b) and 3(c), however, we may find two distinct states \mathbf{p}_2 and \mathbf{p}_5 for coexistences of two independent species and five species, respectively. For low mobility regimes, as we already know in Ref. 33, the coexistence of five species is witnessed [Fig. 3(c)]. As mobility is increased, however, the survival probability of five species gets decreased and the biodiversity is hampered as mobility exceeds the critical mobility. Instead, as shown in Fig. 3(b), two independent

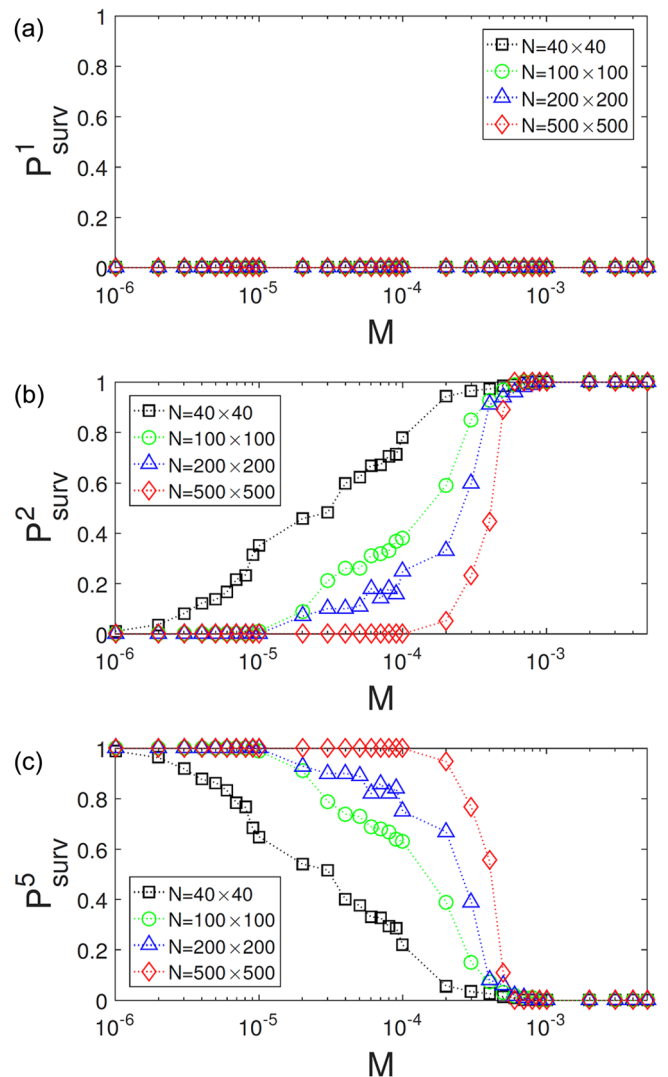


FIG. 3. The survival probability of the ERPS game on a lattice depending on mobilities under the symmetric initial condition. (a) P_{surv}^1 for \mathbf{p}_1 is always zero for all mobility values. In this model, there is no extinction even though mobility is induced. (b) P_{surv}^2 for \mathbf{p}_2 increases as mobility increases. As the lattice size gets larger, a curve rapidly increases and approaches to 1. Thus, two independent species finally survive robustly at high mobility regimes. (c) P_{surv}^5 for \mathbf{p}_5 , on the contrary, gets decreased as mobility increases. Coexistence of all species at low mobility regimes is broken as mobility exceeds the critical mobility.

species may survive robustly for high mobility values. The survival probability P_{surv}^2 sharply increases and approaches 1 as mobility increases for a finite evolution time. From the information of survival probabilities, we may find that states \mathbf{p}_2 and \mathbf{p}_5 can be revealed and the state \mathbf{p}_2 is robust at high mobility regimes.

B. Initial conditions with equally fixed two independent species

When the initial densities of two species in neighbors are fixed, all five \mathbf{p}_2 basins are represented in the triangular phase space Δ_{ABC} for instance. Further, a \mathbf{p}_5 basin can also emerge when low mobility values satisfying $M < M_c$ are considered. As referred in the RPS game,⁴⁶ five \mathbf{p}_2 basins are formed spirally and the coexistence basin is captured and expanded from the point at which all five \mathbf{p}_2 basins meet. For the second consideration of the initial construction defined by Δ_{ACE} , however, basin structures can be obtained in different ways though we consider the same system. Similarly to the case of Δ_{ABC} , the sum of three species will be fixed by a certain value, i.e., $n_a + n_c + n_e = \alpha$ where α will be 0.4, 0.6, and 0.8. Thus, for in each case, the initial densities of species B and D are equally given as 0.3, 0.2, and 0.1, respectively.

Figure 4 shows basin structures in the phase space Δ_{ACE} for different mobilities. On the contrary to the case of Δ_{ABC} as shown in Fig. 1, we find the totally different basin structures by changing initial constructions even when the same system is considered. Top panels in Fig. 4 show basin structures for $\alpha = 0.4$. In this case, we find only three states of \mathbf{p}_2 for $M > M_c$. As M decreases satisfying $M < M_c$, a new basin of

the state \mathbf{p}_2 emerges inside the basin of \mathbf{p}_2 for species C and E (yellow area) as shown in Figs. 4(c), 4(h), and 4(m), which guarantees the survival of species B and E. Simultaneously, the area of yellow and red colored \mathbf{p}_2 basins gets larger and maintains its size as M decreases. As M decreases further, the region inside the yellow \mathbf{p}_2 basin gets fuzzy, which means that all possible five cases of the state \mathbf{p}_2 can occur sensitively depending on the initial densities of species [Figs. 4(c)–4(e)]. For all possible mobility values, the coexistence state \mathbf{p}_5 does not appear in this case. In fact, the coexistence of five species does not occur when $\alpha = 0.4$ and it is obviously validated by examining the survival probability of the state \mathbf{p}_5 (P_{surv}^5) for low mobility values as shown in Fig. 5.

When $\alpha = 0.6$ and 0.8, however, we find all possible five basins of the state \mathbf{p}_2 for high mobility values $M > M_c$, which are illustrated in Figs. 4(f)–4(g) and 4(k)–4(l), respectively. Further, the coexistence \mathbf{p}_5 basin emerges as M decreases satisfying $M < M_c$, which can occur near the concurrence among five \mathbf{p}_2 basins. The change of initial construction thus can lead to the different survival states and the weak spirally entangled basin structures may occur if all basins of the state \mathbf{p}_2 exist.

IV. PREDICTION OF BASINS FOR THE HIGH MOBILITY REGIME BASED ON THE MEAN-FIELD ANALYSIS

The basin structure for the state \mathbf{p}_2 can be predicted analytically since cyclic dynamics can be interpreted in the mean-field framework if M is sufficiently high satisfying $M > M_c$.^{3,46,48}

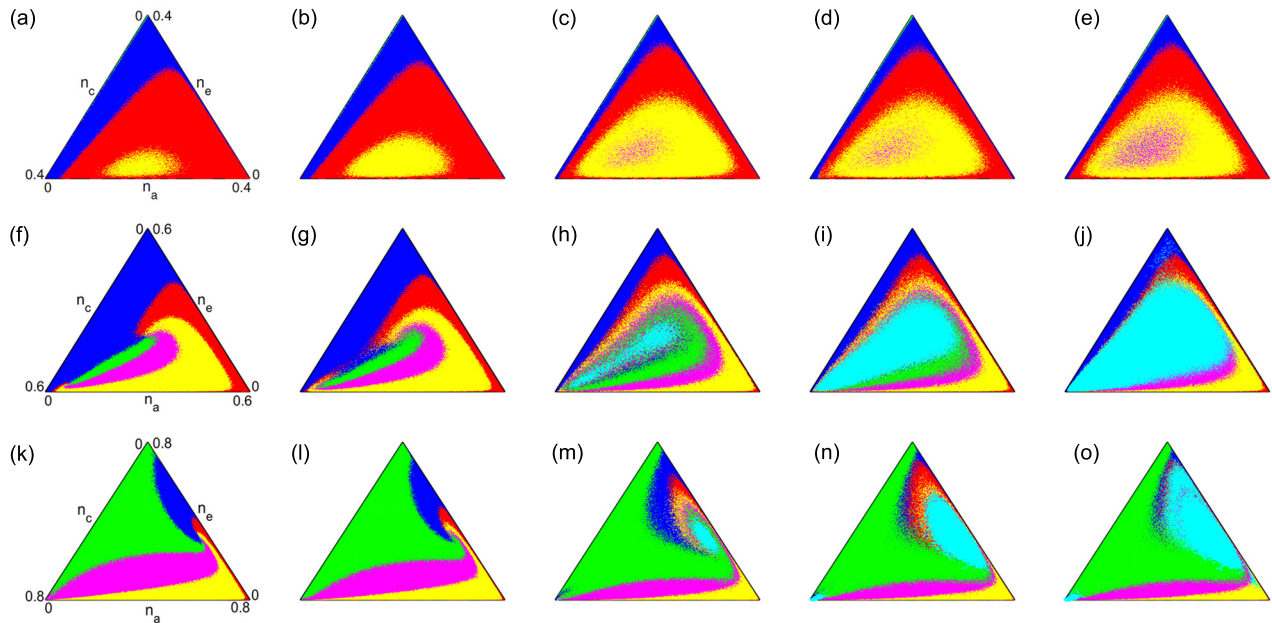


FIG. 4. Basins of final states of the ERPS game represented on the phase space Δ_{ACE} for different values of the mobility M from stochastic simulations. The coordinates denote the initial densities of the three species A, C, and E satisfying $n_a(0) + n_c(0) + n_e(0) = 0.4$ for (a)–(e), 0.6 for (f)–(j), and 0.8 for (k)–(o). All other conditions, including mobilities for simulations are the same in the case Δ_{ABC} . For $\alpha = 0.4$, three cases of \mathbf{p}_2 basins mainly dominate the phase space for $M > M_c$ as shown in (a) and (b). As M is decreased satisfying $M < M_c$, a panel (c), another \mathbf{p}_2 basin emerges inside the region of present \mathbf{p}_2 basin. For more lower mobility values, panels (d) and (e), the region inside one of \mathbf{p}_2 basins gets fuzzy, which means three possible cases of the state \mathbf{p}_2 can be appeared. In this case, the coexistence \mathbf{p}_5 and extinction \mathbf{p}_1 basins are not emerged. For $\alpha = 0.6$ and 0.8, respectively, the phase space is occupied by five \mathbf{p}_2 basins for high mobility values. In this case, these five basins are weak spirally and asymmetrically entangled. On the contrary to $\alpha = 0.4$, the coexistence \mathbf{p}_5 basin exists as M is decreased. As above, the extinction basin does not occur for all possible initial considerations.

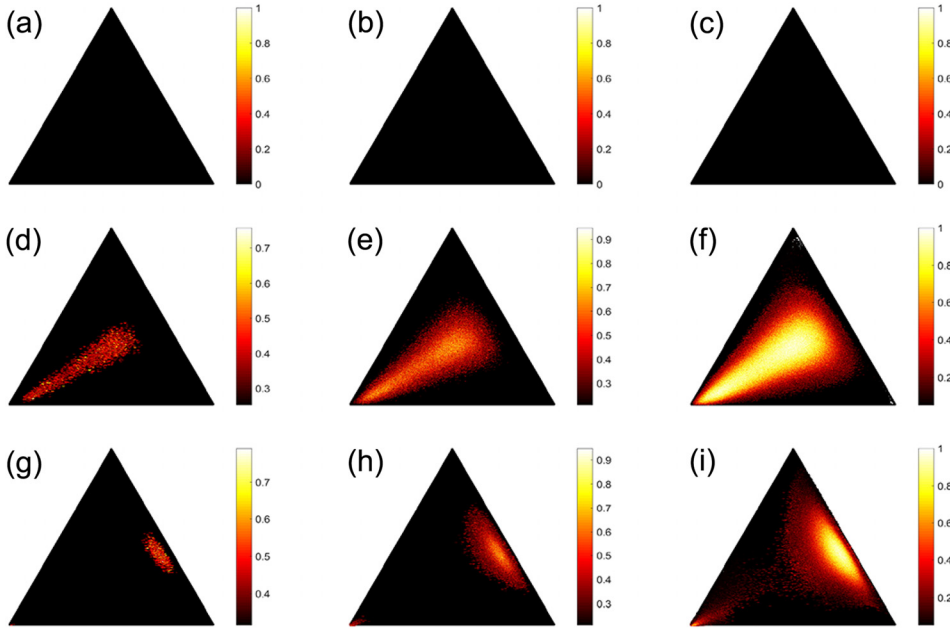


FIG. 5. The survival probability of \mathbf{p}_5 , P_{surv}^5 , for each α for low mobility values M . Mobility values are 10^{-4} for (a), (d), and (g), 10^{-5} for (b), (e), and (h), and 10^{-6} for (c), (f), and (i). Panels from top to bottom indicate the different value α as 0.4, 0.6, and 0.8, respectively. As we show in the basin structure in Fig. 4, the survival probability of the state \mathbf{p}_5 is always zero which means the coexistence of five species does not exist for $\alpha = 0.4$ even though mobility values are low.

$$\begin{cases} \dot{n}_a = \mu n_a(1 - \rho) - \sigma n_a n_e, \\ \dot{n}_b = \mu n_b(1 - \rho) - \sigma n_b n_a, \\ \dot{n}_c = \mu n_c(1 - \rho) - \sigma n_c n_b, \\ \dot{n}_d = \mu n_d(1 - \rho) - \sigma n_d n_c, \\ \dot{n}_e = \mu n_e(1 - \rho) - \sigma n_e n_d, \end{cases} \quad (4)$$

where $\rho = n_a + n_b + n_c + n_d + n_e$ is the total density at time t . From direct calculations by setting $\dot{n}_a = \dot{n}_b = \dot{n}_c = \dot{n}_d = \dot{n}_e = 0$, three types of equilibrium points can be obtained.

For the first type \mathbf{p}_1 which stands for the extinction state, we have five points: $(1, 0, 0, 0, 0)$, $(0, 1, 0, 0, 0)$, $(0, 0, 1, 0, 0)$, $(0, 0, 0, 1, 0)$, and $(0, 0, 0, 0, 1)$. For the second type \mathbf{p}_2 which indicates the coexistence of two species, the system allows five points: $(a^*, 0, c^*, 0, 0)$, $(a^*, 0, 0, d^*, 0)$, $(0, b^*, 0, d^*, 0)$, $(0, b^*, 0, 0, e^*)$, and $(0, 0, c^*, 0, e^*)$ as illustrated in Fig. 2. These five points are not defined by specific forms, but densities of existing two species satisfy $n_x + n_y = 1$. The last type \mathbf{p}_5 for the coexistence of all species is defined by $\mu/(5\mu + \sigma)(1, 1, 1, 1, 1)$. Mathematically, Eq. (4) cannot yield absolute states \mathbf{p}_1 , \mathbf{p}_2 , and \mathbf{p}_5 since these all

equilibrium points are unstable by the linear stability analysis. Nevertheless, Eq. (4) always shows the existence of two species which are independent of interspecific competitions due to existence of heteroclinic orbits among five points of \mathbf{p}_2 .^{3,49} However, Eq. (4) can still identify the basin structures by considering the physical meaning of existence that the number of existed species is less than unity due to the discrete nature of individuals.⁴⁶ To make an unbiased comparison of basin structures between the lattice simulations and theoretical predictions, we calculate the final states of Eq. (4) basins for all possible initial conditions under the same considerations as lattice simulations.

In Eq. (4), the configuration among states \mathbf{p}_1 and \mathbf{p}_2 is ambiguous due to the roundoff error on computations since equilibrium points of \mathbf{p}_2 are defined by $n_x + n_y = 1$. To handle this problem, we consider the threshold of densities by $err = 10^{-6}$, which means that if density from the numerical computation is calculated satisfying $n_i < err$, then we regard the density of the species as zero.

Figures 6 and 7 show the comparison of basin structures of the state \mathbf{p}_2 between lattice simulations for a high mobility

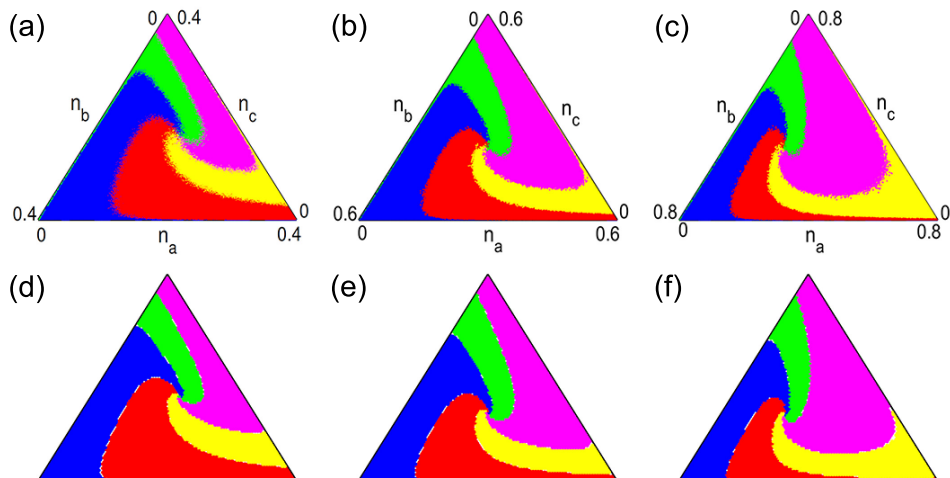


FIG. 6. Basin structures for \mathbf{p}_2 on Δ_{ABC} derived from lattice simulations (tops) with a mobility $M = 3 \times 10^{-3}$ and theoretical predictions from Eq. (4) (bottoms). The sum α of initial densities of three species are 0.4, 0.6, and 0.8 from left to right, respectively. Basins from theoretical predictions are also spirally entangled and all cases of the state \mathbf{p}_2 appear which match lattice simulations for each α .

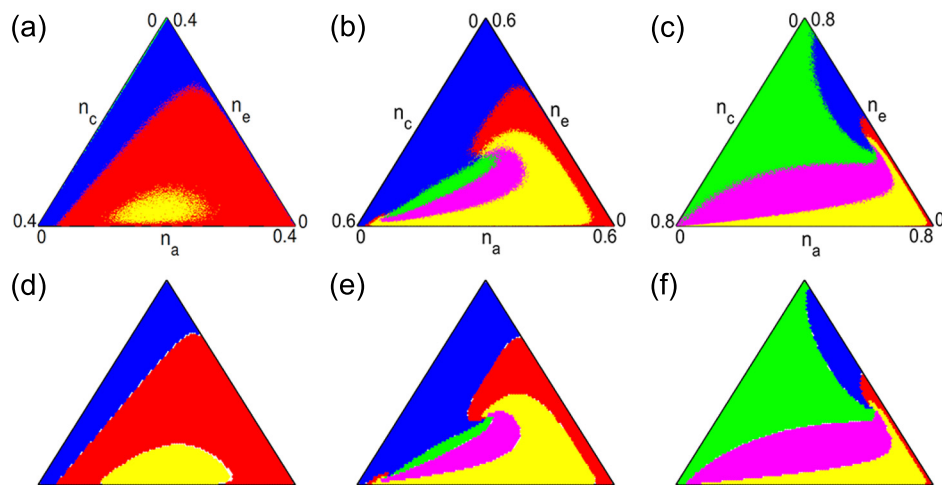


FIG. 7. Basin structures for \mathbf{p}_2 on Δ_{ACE} derived from lattice simulations (tops) with a mobility $M = 3 \times 10^{-3}$ and theoretical predictions from Eq. (4) (bottoms). The sum of initial densities of three species is the same in the case of Δ_{ABC} . Basin structures from lattice simulations are matched with theoretical predictions for each α .

value and theoretical predictions on two different initial phase spaces Δ_{ABC} and Δ_{ACE} , respectively. In each figure, the sum α of initial densities of three species is given 0.4, 0.6, and 0.8 for (a) and (d), (b) and (e), and (c) and (f), respectively. The initial densities of two other species are kept uniform as above.

As shown in Figs. 6 and 7, solutions of Eq. (4) for all possible initial considerations agree obviously with the results from lattice simulation for high mobility values. The coordinates of points in the phase space represent all possible initial conditions and thus solutions of Eq. (4) may sensitively depend on initial conditions, especially near the point where five \mathbf{p}_2 basins meet. Thus, the theoretical prediction of Eq. (4) may support our numerical results from extensive lattice simulations for high mobility values.

V. CONCLUSION

As the one of the key factors to maintain the biodiversity, in conclusion, we have addressed the effect of initial densities by investigating basins of different survival states in the cyclically competing mobile five species game on a lattice. Here, basin structures are characterized by the final state in the triangular phase space. We have found that for the extended rock-paper-scissors game, basin structures can be changed depending on the initial considerations. To be concrete, two types of initial phase spaces have been considered. When the initial space contains three neighboring species, e.g., Δ_{ABC} , five basins of the two species survival states spirally entangle. About the point that five basins meet, a coexistence basin can emerge, depending on the mobility parameter of the fundamental spatiotemporal dynamical system.

Although the same system is considered, however, the basin structures are completely changed if the initial space is considered in a different way. When the initial space contains two relative and one independent species, e.g., Δ_{ACE} , all possible basins of the two species survival states can exist or not due to the total amount of initial densities of three species. For all possible initial considerations, the extinction basin never arises which has been guaranteed by measuring the survival probability and the non-existing basin area of

\mathbf{p}_1 . Similar to the basin stability,⁴³ asymptotic states of the system have been obviously depicted by the area of basins of distinct states. Stochastic simulations for high mobility regimes on a microscopic level are consistent with the theoretical prediction on a macroscopic level and we found that five points constituting a stable heteroclinic cycle appear from lattice simulations as a robust feature. Through our microscopic and macroscopic endeavors, we suggest the persuasive insight for the biodiversity associated with the extended rock-paper-scissors game which is sensitively affected by the initial densities of species.

ACKNOWLEDGMENTS

J.P. was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (No. NRF-2016R1C1B1010340).

- ¹R. M. May, *Stability and Complexity in Model Ecosystems* (Princeton University Press, Princeton, NJ, 1973).
- ²R. M. May, *Science* **186**, 645 (1974).
- ³R. M. May and W. J. Leonard, *SIAM J. Appl. Math.* **29**, 243 (1975).
- ⁴G. Szabó and G. Fáth, *Phys. Rep.* **446**, 97 (2007).
- ⁵M. Perc and A. Szolnoki, *New J. Phys.* **9**, 267 (2007).
- ⁶J. Hofbauer and K. Sigmund, *Evolutionary Games and Population Dynamics* (Cambridge University Press, Cambridge, 1998).
- ⁷B. Sinervo and C. M. Lively, *Nature* **380**, 240 (1996).
- ⁸T. L. Czárán, R. F. Hoekstra, and L. Pagie, *Proc. Natl. Acad. Sci. U.S.A.* **99**, 786 (2002).
- ⁹C. E. Paquin and J. Adams, *Nature* **306**, 368 (1983).
- ¹⁰B. Kerr, M. A. Riley, M. W. Feldman, and B. J. M. Bohannan, *Nature* **418**, 171 (2002).
- ¹¹T. Reichenbach, M. Mobilia, and E. Frey, *Nature* **448**, 1046 (2007).
- ¹²T. Reichenbach, M. Mobilia, and E. Frey, *J. Theor. Biol.* **254**, 368 (2008).
- ¹³A. Traulsen, J. C. Claussen, and C. Hauert, *Phys. Rev. Lett.* **95**, 238701 (2005).
- ¹⁴A. Traulsen, J. C. Claussen, and C. Hauert, *Phys. Rev. E* **74**, 011901 (2006).
- ¹⁵A. P. O. Müller and J. A. C. Gallas, *Phys. Rev. E* **82**, 052901 (2010).
- ¹⁶J. C. Claussen and A. Traulsen, *Phys. Rev. Lett.* **100**, 058104 (2008).
- ¹⁷W.-X. Wang, X. Ni, Y.-C. Lai, and C. Grebogi, *Phys. Rev. E* **83**, 011917 (2011).
- ¹⁸D. Hua, L. Dai, and C. Lin, *Europhys. Lett.* **101**, 38004 (2013).
- ¹⁹Q. He, M. Mobilia, and U. C. Täuber, *Phys. Rev. E* **82**, 051909 (2010).
- ²⁰Q. He, M. Mobilia, and U. C. Täuber, *Eur. Phys. J. B* **82**, 97 (2011).
- ²¹J. Park, Y. Do, Z.-G. Huang, and Y.-C. Lai, *Chaos* **23**, 023128 (2013).
- ²²R. Yang, W.-X. Wang, Y.-C. Lai, and C. Grebogi, *Chaos* **20**, 023113 (2010).
- ²³G. Szabó, A. Szolnoki, and I. Borsos, *Phys. Rev. E* **77**, 041919 (2008).

- ²⁴L. Frachebourg, P. L. Krapivsky, and E. Ben-Naim, *Phys. Rev. E* **54**, 6186 (1996).
- ²⁵S. O. Case, C. H. Durney, M. Pleimling, and R. K. P. Zia, *Europhys. Lett.* **92**, 58003 (2010).
- ²⁶K. Sato, N. Yoshida, and N. Konno, *Appl. Math. Comput.* **126**, 255 (2002).
- ²⁷J. Knebel, T. Krüger, M. F. Weber, and E. Frey, *Phys. Rev. Lett.* **110**, 168106 (2013).
- ²⁸P. P. Avelino, D. Bazeia, L. Losano, and J. Menezes, *Phys. Rev. E* **86**, 031119 (2012).
- ²⁹P. P. Avelino, D. Bazeia, L. Losano, J. Menezes, and B. F. Oliveira, *Phys. Rev. E* **86**, 036112 (2012).
- ³⁰J. Vandermeer and S. Yitbarek, *J. Theor. Biol.* **300**, 48 (2012).
- ³¹Y. Kang, Q. Pan, X. Wang, and M. He, *Physica A* **392**, 2652 (2013).
- ³²J. Vukov, A. Szolnoki, and G. Szabó, *Phys. Rev. E* **88**, 022123 (2013).
- ³³S.-S. Feng and C.-C. Qiang, *Physica A* **392**, 4675 (2013).
- ³⁴H. Cheng, N. Yao, Z.-G. Huang, J. Park, Y. Do, and Y.-C. Lai, *Sci. Rep.* **4**, 7486 (2014).
- ³⁵Y. Kang, Q. Pan, X. Wang, and M. He, *PLoS One* **11**, e0157938 (2016).
- ³⁶E. Glasner and B. Weiss, *Nonlinearity* **6**, 1067 (1993).
- ³⁷J. Urías, R. Rechtman, and A. Enciso, *Chaos* **7**, 688 (1997).
- ³⁸Y. Wang, G. Wei, and W. H. Campbell, *Topol. Appl.* **156**, 803 (2009).
- ³⁹J.-P. Eckmann and D. Ruelle, *Rev. Mod. Phys.* **57**, 617 (1985).
- ⁴⁰T. Kapitaniak, *Chaos, Solitons Fractals* **2**, 519 (1992).
- ⁴¹T. Kapitaniak, *Chaos, Solitons Fractals* **6**, 237 (1995).
- ⁴²B. R. Andrievskii and A. L. Fradkov, *Autom. Remote Control* **64**, 673 (2003).
- ⁴³P. J. Menck, J. Heitzig, N. Marwan, and J. Kurths, *Nat. Phys.* **9**, 89 (2013).
- ⁴⁴J. Huisman and F. J. Weissing, *Nature* **402**, 407 (1999).
- ⁴⁵J. Huisman and F. J. Weissing, *Am. Nat.* **157**, 488 (2001).
- ⁴⁶H. Shi, W.-X. Wang, R. Yang, and Y.-C. Lai, *Phys. Rev. E* **81**, 030901(R) (2010).
- ⁴⁷X. Ni, R. Yang, W.-X. Wang, Y.-C. Lai, and C. Grebogi, *Chaos* **20**, 045116 (2010).
- ⁴⁸R. Durrett and S. Levin, *Theor. Popul. Biol.* **46**, 363 (1994).
- ⁴⁹P. M. Altrock and A. Traulsen, *Phys. Rev. E* **80**, 011909 (2009).
- ⁵⁰S. Redner, *A Guide to First-Passage Processes* (Cambridge University Press, Cambridge, 2001).