Instructional Science
https://doi.org/10.1007/s11251-023-09654-4

ORIGINAL RESEARCH

Exploring when learners become aware of their knowledge gaps: Content analyses of learner discussions

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Received: 30 October 2021 / Accepted: 12 October 2023

Abstract

This study investigates when and how awareness of knowledge gaps (AKG) manifests by observing the problem-solving phase of the educational approach known as problem-solving followed by instruction (PS-I). By comprehensively exploring cognitive and metacognitive process of learners during this phase and categorizing students' judgements of knowledge structure in relation to AKG, it strengthens the underlying mechanisms of PS-I. With sixteen university students as participants, this study quantitatively and qualitatively analyzes conversations that take place during problem-solving activities. In the analysis, the authors suggest a total of ten cognitive and metacognitive events that occur and six judgements of knowledge structure in relation to AKG. The findings indicate that students spend most of their time solving the problem and seldom evaluate their thoughts; few express awareness of a knowledge gap. The authors discuss the relationships between the judgements of knowledge structure and consider when—and to what extent—students perceive their knowledge gaps. Lastly, the authors bring four learning behaviors (i.e., representing and reflecting on knowledge; recognizing and specifying knowledge gaps) with possible instructional strategies to promote each learning behavior.

Keywords Awareness of knowledge gaps · Collaborative learning · Content analysis · Problem-solving followed by instruction

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Published online: 16 December 2023

Introduction

A substantial body of research into the processes and outcomes of the recognition of incorrect, incoherent, or insufficient knowledge (i.e., awareness of knowledge gaps, hereafter abbreviated as AKG) and of constructing or repairing knowledge structures in light of this awareness has been conducted in the disciplines of cognitive science and educational psychology. Among the foundational theories in this field are failure-driven memory (Schank, 1999), impasse-driven learning (VanLehn, 1988), and self-repair (or revision) of imperfect mental models (Chi, 2000). A common underlying mechanism posited by these theories is that once individuals recognize insufficiency, incoherence, and conflicts within their knowledge in the face of an impasse, they attempt to close these gaps by revising their current knowledge (unless they ignore or reject the gaps) and thereby improve their knowledge structure or schema. More recently, this classic paradigm of cognitive development in learning sciences has been applied to instructional approaches to failure-based learning.

Problem-solving followed by instruction (PS-I), which typically includes instructional methods such as productive failure (e.g., Kapur, 2016) and invention activity (Schwartz & Martin, 2004), is one of the most salient areas of research into failure-based learning. In fact, Loibl et al. (2017) reviewed different applications of a PS-I approach and highlighted the mechanism of AKG as a core process that results in repaired and improved knowledge when experienced with preceding and succeeding (meta-)cognitive processes (i.e., a sequential loop of prior knowledge activation, awareness of knowledge gaps, and recognition of the deep features of the target concept). In a failure-based, problem-solving context like PS-I, failing to solve a problem is unproductive if students are unaware of their failure, as learners need to *know what they don't know* to engage (meta-)cognitively and affectively with emerging learning opportunities. That is, becoming aware of knowledge gaps subsequently allows students to reassess the problem situation, try new or repair known solution methods, reflect on their current knowledge structure in relation to new knowledge, become motivated to pay attention to previously identified knowledge gaps, and fill these knowledge gaps through in-depth processing of instructional explanations (Hmelo-Silver et al., 2018). These cognitive, metacognitive, and affective experiences that shape AKG are thus what propels the process of making failure productive in a PS-I approach.

For AKG to arise, certain processes of cognitive and metacognitive actions should occur in advance. Generally, students are understood to experience AKG when they encounter procedural impasse (VanLehn, 1999) or cognitive conflicts (Chi, 2000) during problem-solving, processing contrasting cases or comparing solutions. In the literature on curiosity, AKG is viewed as a product of both context-based and information-based prediction errors (Gruber & Ranganath, 2019). That is, individuals become aware of their knowledge gaps when expectations based on past experiences conflict with the current situation, or when they detect a gap in previously understood information on a particular topic. However, further specific descriptions of the cognitive and metacognitive *processes* that produces AKG are needed to supplement these causal relations. Some students may notice procedural bugs and cognitive conflicts and grapple with closing the gaps between their current understanding and the knowledge to be explored, with or without specifying the knowledge gaps, while others fail to recognize the gaps or simply ignore them. In fact, individual differences in AKG are quite common, but rarely systematically explicated in terms of process. A small number of studies have addressed the existence and impacts of these differences in terms of their perceived level as measured by a self-report scale (Gloger-Frey et al., 2015) and its degree of gap specificity (Loibl & Rummel, 2014). Interestingly, a recent study by

Trninic et al. (2022) demonstrated that, despite significant variations in Knowledge Gap Awareness measures across different preparatory conditions, there was no corresponding difference in learning outcomes. This finding challenges the simplified notion that awareness of knowledge gaps invariably leads to better learning and underscores the need for further investigation into the intricate processes involved. However, while these findings highlight the complexity of the relation between Knowledge Gap Awareness and learning outcomes, they do not delve into the reasons for why some students notice potential knowledge gaps but fail to acknowledge and cope with them. Chi (2000) also emphasized that an individual may not repair their mental model when they misinterpret or reject conflicting information. Consequently, the question this study seeks to address remains unanswered: *when*—i.e., under what circumstances—do students, either intentionally or unintentionally, reject, ignore, or misinterpret their perceived conflicts while others take additional action to resolve them?

Theoretical background

Previous research on awareness of knowledge gap

Earlier research from various fields, such as psychology, educational psychology and learning sciences, has examined various dimensions of AKG: subject, level, accuracy, and specificity (see Table 1). This section discusses how AKG has been examined within and outside the PS-I literature and regarding its different dimensions. Outside the context of PS-I, a research strand about the 'feeling-of-knowing' (e.g., Hart, 1965) has examined the level and accuracy of metacognitive judgment of one's available knowledge. This is usually described as the 'tip-of-the-tongue' phenomenon, where one thinks one knows something but fails to completely retrieve it from one's memory (Koriat, 1998). Litman et al. (2005) further classified different states of the "feeling-of-knowing" regarding its level: "Don't Know" (high level) and "I Know" (none or low level) in addition to "Tip-Of-Tongue" (moderate level). "Tip-Of-Tongue" is associated with curiosity and exploratory behaviors, while the "feeling-of-not-knowing" ("Don't Know") corresponds to less curiosity and exploration, and "I Know" is associated with the least curiosity. The other predominant research strand is judgment of learning, which underscores the significance of the accuracy of AKG**.** Judgement of learning refers to a self-monitored or predicted state that one can perform during an upcoming exam (Dunlosky & Nelson, 1992). The accuracy of

Table 1 Research dimensions of AKG

Dimension	Features
Subject	The domain where gaps in knowledge (structure) occur, e.g., procedural bugs, conceptual conflicts, violated predictions
Level	The extent of an individual's recognition of the gaps, e.g., high or low level of awareness of knowledge gaps (assessed through Likert scale)
Accuracy	The extent to which an individual correctly perceives their existing knowledge gaps, e.g., misinterpretation of failures, misperception of understanding, or judgement of learning
Specificity	The degree to which an individual specifies the gaps, e.g., global awareness of knowledge gaps (feeling-of-(not)-knowing) and awareness of specified knowledge gaps

judgement of learning is critical because the judgement can "directly influence what people choose to study" (Metcalfe & Finn, 2008, p. 174) and has a "guiding role in the self-paced acquisition of new information" (Nelson & Dunlosky, 1991, p. 267). Both concepts (feeling-of-knowing and judgement of learning) illustrate that how *much* and *accurately* people think they know–metacognitive judgment–is closely related to their motivation in upcoming learning events.

In PS-I literatures, AKG is often regarded as a beneficial learning mechanism of the PS-I model and has been used to provide evidence for its core function. Researchers interested in invention activities (see Schwartz & Martin, 2004) first conceptualized knowledge gaps and measured the level of AKG with questionnaires. Glogger-Frey et al. (2015) focused on experiencing knowledge gaps as a cognitive outcome of invention activities and assessed it through nine self-rated items with six-point scales (e.g., 'My knowledge was insufficient to complete the task'; see Table 1). Their results indicated that invention activity group reported higher level of knowledge gaps than worked-out example group. Newman and DeCaro (2019) emphasized AKG as a metacognitive benefit of exploratory learning and guiding students to perceive knowledge gaps. Using a self-reporting scale developed in the context of assessing consumers' knowledge in business contexts (Flynn & Goldsmith, 1999), they examined whether students perceive knowledge gaps at different levels based on the order of instruction (i.e., instruction-first vs. explore-first) or the activity (i.e., invention vs. worked example). Finding evidence of a significant main effect and an interaction effect, they suggested providing pre-tests with worked examples as a strategy that supports AKG.

There have also been meaningful findings about the different degrees of specificity of AKG and methods that might increase it. Loibl and Rummel (2014) distinguished between global awareness of knowledge gaps (i.e., 'awareness that they have knowledge gaps without being able to specify which component they are lacking', p. 75) and awareness of specified knowledge gaps (i.e., awareness elicited during instruction by 'helping them to detect differences in a more specific manner', p. 75). They argued that identifying knowledge gaps may induce modifications to partial, naïve, or erroneous schemas and suggested that instruction that compares a student's solutions to the canonical solution can help specify the gaps present in the problem-solving phase. Using five items (e.g., 'I lack knowledge required to solve this problem'; 'I think I did not find a canonical solution for this problem', p. 81), they demonstrated that the PS-I approach can trigger global AKG, with students in the PS-I group reporting more knowledge gaps than those in the I-PS group. In their follow-up studies, they emphasized compare/contrast activities as a way to specify knowledge gaps and connect to the next level of knowledge (i.e., deep feature recognition) (Loibl et al., 2017, 2020).

Despite the extensive PS-I research on AKG (see Table 1), little is known about how student experiences of AKG differ in the same learning situation. This entails not only understanding the different levels and specificities of AKG but also whether and how knowledge gaps are recognized. Researchers have reported that some students may ignore uncertainty (Jordan & McDaniel, 2014) or make inaccurate judgments of their knowledge states (Rawson & Dunlosky, 2007) and thus lose the opportunity to address their deficiencies. For example, Nachtigall et al. (2020) measured learners' perceived competence as an indicator of their awareness of knowledge limitations and found that a PS-I (productive failure) group reported higher perceived competence than a direct instruction group after the first learning phase (a problem-solving phase for the PS-I group and an instruction phase for the direct instruction group). The conclusion here is that learners under PS-I conditions may not develop an appropriate awareness of their knowledge limitations. Nevertheless, it is often the case that students feel that their knowledge is incomplete when experiencing PS-I; however, this feeling-of-not-knowing in a broader sense should be distinguished from the phenomenon of specifying the knowledge they are lacking, which often requires instructional support (e.g., comparing their solutions or mental models to those of others; Gadgil et al., 2012; Loibl & Rummel, 2014).

Problem-solving processes

A thorough analysis of students' problem-solving processes allows us to gain a better understanding of the circumstances in which students either reject, ignore, or misinterpret their perceived knowledge gaps in the PS-I context. Having a model of the problem-solving process in PS-I can greatly assist in analyzing and comprehending the experiences students have during problem-solving. In this section, we will explore various models of the problem-solving process that researchers have proposed. The examination of these models will subsequently contribute to the development of a coding scheme employed in this paper for the analysis.

Several educational researchers have developed models of problem-solving processes. The models are similar in their inclusion of certain cognitive and metacognitive activities that typically occur during problem-solving but are otherwise diverse in their foci and contexts. The two common characteristics that most models share are (a) the assumption that the process of problem-solving differs between experts and novices and (b) that problem-solving involves temporal sequencing and cyclical processes of cognitive and metacognitive activity (Grave et al., 1996; Hartmann et al., 2022; Hmelo-Silver, 2004; Jonassen, 1997; Pretz et al., 2003). With regard to the first characteristic, the novice student, unlike the expert, is not equipped with a problem schema that can adequately generate and implement a solution. Thus, students rely on generalized problem-solving strategies, such as means-ends analysis, problem-analysis, problem decomposition and setting sub-goals (Hartmann et al., 2022; Hmelo-Silver, 2004; Jonassen, 1997; Owen & Sweller, 1985). Once students devise a potential solution, they may evaluate their problem-solving processes and outcomes using acquired abstract knowledge (Hmelo-Silver, 2004). If they find that the solutions are incorrect or inappropriate, they redefine and re-analyze the problems, then regenerate and re-evaluate alternate solutions until they conclude with at least seemingly valid outcomes (Jonassen, 1997). After they complete the process, they can also identify a new problem with their new understanding, and may engage with another cycle of problem-solving (Pretz et al., 2003).

In addition to these two shared understandings of problem-solving processes among novices, several models have proposed different processes and pathways. For example, IDEAL (Identify, Define, Explore, Act, and Look back; Bransford & Stein, 1993), is widely utilized as a problem-solving model. Another popular model is that of Sternberg (1986) and Pretz et al. (2003) which features meta-level activities for planning, monitoring, and evaluating problem-solving processes. Synthesizing previous research, Pretz and colleagues suggest a cyclical model of recognizing a problem, defining and representing it mentally, developing a solution strategy, organizing one's knowledge about the problem, allocating mental and physical resources, monitoring one's progress toward the goal, and evaluating the solution for accuracy. Building upon these classic models, Gick (1986) and Jonassen (1997) simplified the model into a schematic process: problem representation, a search for a solution, and its implementation (PS-I). According to this process, when a problem-solver (often a novice) fails to activate a problem schema, they start to search for a solution through generalized strategies, such as analogical reasoning, means-ends analysis, and setting sub-goals by simplifying the problem.

These models highlight the nature of the problem-solving process, suggesting that it continues to iterate, especially when students are not satisfied with their initial attempts, until a solution is found. However, they elaborate neither how problem-solvers reflect on their knowledge or solutions, nor how they react, manage, or reduce the existing gaps between their solutions and goal states. In this regard, Hmelo-Silver (2004) emphasized one of the characteristics of problem-based learning (PBL) and suggested a problem-based learning cycle (or a PBL tutorial process): presenting a problem scenario, identifying relevant facts for problem-solving, generating assumptions (or hypotheses) for possible solutions, identifying knowledge gaps (or deficiencies), applying newly gained knowledge and evaluating its alignment with initial hypotheses. Within this model, it is important that students identify both what they do and do not know and what they should know. Similarly, Grave et al. (1996) highlighted that identifying knowledge gaps should be followed by individual study, which then encourages students to re-evaluate and re-analyze the problem, challenge and question their existing knowledge, build alternate solutions, and refine their understandings. That is, after recognizing gaps in their knowledge, a student may be more likely to engage with deep cognitive and metacognitive processes of modifying and changing their schema.

Based on an understanding of students' cognitive and metacognitive activities during trial-and-error problem-solving processes, researchers developed several coding schemes to analyze the ways in which students struggle or fail to solve a problem and try to overcome it. For example, Große and Renkl (2007) suggested a coding scheme that can be applied to the analysis of a student's worked examples of problem-solving, which focuses on their recognition of impasse, attempts at repair, and reflection on errors. Similarly, Grig and Benson (2014), in an engineering problem-solving context, developed a coding scheme that highlights students' recognition and monitoring of conceptual and procedural errors as well as the evaluation and revision of potential solutions.

Examining problem-solving processes specifically in the context of productive failure, Kapur and Kinzer (2009) suggested a coding scheme adapted from the interaction analysis developed by Poole and Holmes (1995), using the following terms: problem analysis, problem critique, orientation, criteria development, solution development, solution evaluation, and non-task. A related scheme (structured through task definition, analysis, planning and design, implementation, and evaluation) was developed by Roll et al. (2012) in the context of so-called "invention activities". However, these characterizations of problem-solving processes do not readily lend themselves to research which focuses on (meta-)cognitive activities in trial-and-error contexts because they do not necessarily classify cognitive and metacognitive processes of recognizing and repairing impasse or conflicts. Most recently, Hartmann et al. (2022) suggested a coding scheme that highlights productive and unproductive problem-solving patterns. Like those of Große and Renkl (2007) and Grig and Benson (2014), this scheme distinguishes students' problem-solving activities where they recognize errors or conflicts and where they identify the reasons why these issues arise (see also Brand et al., 2021 for variations in the context of productive and vicarious failure).

Table 2 summarizes the cognitive and metacognitive actions during the problem-solving process identified from previous research. Three general phases are identified: problem analysis and representation, solution development, and solution evaluation. In the first phase, problem-solvers begin by analyzing problem situations, identifying relevant elements, and representing them in their mental models. Then, they engage in generating

Table 2 Summary of cognitive and metacognitive actions during (observation of) problem-solving across different contexts and focuses

General process	Context (Focus)							
	Problem-solving (Process)	Problem-solving (Meta-level activities)	Problem-solving (Well-structured problem)	Problem-solving (Self-directed learning)	Worked examples (Reflection on errors)	PS-I (Peer interaction)	PS-I (Invention activities)	PS-I (Sequential patterns for productive PS)
Problem analysis and representation	- Identify and define problems	- Recognizing or identifying the problem - Defining and representing the problem mentally	- Problem representation	- Presenting a problem scenario - Identifying relevant facts for problem-solving	- Elaboration of problem situation - Elaboration of problem type	- Problem analysis - Problem critique	- Task definition - Analysis	- Analysis of the goal state
Solution development	- Explore and act on problem-solving strategies	- Developing a solution strategy - Organizing one's knowledge about the problem - Allocating mental and physical resources	- Search for solution - Implement solutions	- Generate assumptions (or hypotheses) for possible solutions	- Impasse - Principle-based explanation - Subgoal - Goal-operator-combination - Anticipative reasoning	- Criteria development - Solution development	- Plan and design - Implementation	- Description of the solution - Execution of the solution - Unsubstantiated conclusion - Substantiated conclusion
Solution evaluation	- Look back	- Monitoring one's progress toward the goal - Evaluating the solution for accuracy		- Identify knowledge gaps (or deficiency), - Applying their newly gained knowledge - Evaluating if it aligns with their hypotheses	- Identifying the error - Reason for the error - Adapted problem formulation - Correct solution	- Solution evaluation	- Evaluation	- Unsubstantiated evaluation - Substantiated evaluation

Table 2 (continued)

General process	Context (Focus)							
	Problem-solving (Process)	Problem-solving (Meta-level activities)	Problem-solving (Well-structured problem)	Problem-solving (Self-directed learning)	Worked examples (Reflection on errors)	PS-I (Peer interaction)	PS-I (Invention activities)	PS-I (Sequential patterns for productive PS)
Related research	Bransford and Stein (1993)	Sternberg (1986) Pretz et al. (2003)	Gick (1986) Jonassen (1997)	Hmelo-Silver (2004) Grave et al. (1996)	Große and Renkl (2007)	Kapur and Kinzer (2009) Poole and Holmes (1995)	Roll et al. (2012)	Hartmann et al. (2022)

potential solutions by setting (sub-)goals, hypotheses and assumptions, planning, developing, and implementing strategies, and generating potential solutions. Lastly, they monitor and evaluate their problem-solving processes, as well as the solutions generated. These three phases may iterate until problem-solvers finalize a potential solution. How this paper developed the coding scheme based on the literature review of problem-solving processes and the three phases will be covered in detail in the Data Analysis section.

The present study

As described above, the cognitive and metacognitive activities that involve evaluation and reflection of problem-solving processes, recognition of errors and conflicts, and identification of their causes, and repair or revision of solutions are highlighted in research on problem-solving. However, even though some recent studies in PS-I have explored cognitive and metacognitive actions that shape AKG experiences, a lack of understanding of (meta-)cognitive processes during problem-solving that contribute to different experiences of AKG impedes further progress in advancing both theoretical understanding and instructional practice in a PS-I approach. Once a better understanding of individual experiences of AKG is established, further research on which (meta-)cognitive processes and actions promote or hinder productive learning can be undertaken, which will facilitate the improvement of instructional strategies.

With this broader goal, this study makes a preliminary attempt (a) to deductively investigate problem-solving process in PS-I contexts, with a particular focus on students' approaches to impasse, and (b) to inductively explore how students judge their knowledge structures, which may vary depending on the different (meta-)cognitive actions they take. For example, it is plausible that students who engage in finding errors and recognizing cognitive conflicts would be aware of either approximate or specific knowledge gaps, whereas students who reject or ignore perceived knowledge gaps would be likely to be in the state of "feeling-of-not-knowing" (i.e., global AKG). However, the full picture of these different pathways of judging one's knowledge structures and different AKG experiences remains unclear. As mentioned earlier, the problem-solving process has been extensively addressed by several empirical studies within and outside PS-I contexts. Nevertheless, it is useful to conduct another deductive analysis of students' cognitive and metacognitive actions, focusing on how students' problem-solving processes may unfold and progress toward their (non-)recognition of knowledge gaps. This new approach will have analytical implications for subsequent inductive explorations of how students judge their knowledge structures. By investigating perceived knowledge structures (that should vary among individuals), analysis can distinguish individual differences of self-judgements of knowledge structure. The combination of these two analyses can generate evidence-based, systematic explanations of when students do and do not recognize their knowledge gaps and which differences in cognitive and metacognitive processes engender varying experiences of AKG. This close examination of cognitive and metacognitive conditions and the concomitant judgements of knowledge structure will generate theoretical implications for PS-I mechanisms that additionally account for the factors that affect AKG.

Importantly, two caveats should be made explicit here. First, even though this study focuses on the problem-solving phase (and not on the phase of instruction) in PS-I, it is not designed to suggest that students experience AKG only in this phase. Given its characteristic of *detecting* something incongruent, AKG should arise in the presence of new information or thoughts. It is *how* and *when* (i.e., in which (meta-)cognitive conditions)

AKG arises that is of interest, not within *which phase* that AKG arises. In fact, instructions and materials play a crucial role in forming and improving AKG experiences, as discussed in several studies that address the effects of comparing student-generated solutions with canonical versions (e.g., Gadgil et al., 2012; Loibl & Rummel, 2014). Nevertheless, in order to understand the nature of the AKG experience of novice students, this research focuses on the problem-solving phase where no such instructional supports are present. Second, as mentioned earlier, this research is an explorative and preliminary attempt to create a broader understanding of the impacts of (meta-)cognitive actions on student learning within an AKG framework and how this knowledge might feed into the design of instructional strategies that lead to better student outcomes. With these caveats, this research posits the following research questions:

1. What cognitive and metacognitive actions do learners manifest during problem-solving in PS-I?
2. Through what cognitive and metacognitive processes do students make judgements about their knowledge structures?

Methodology

This study employed an explanatory sequential mixed methods approach: the authors conducted a quantitative content analysis in a deductive manner and, based on its results, performed a qualitative content analysis in an inductive manner to consolidate the understanding of the relevant (meta-)cognitive actions. The advantage of content analysis lies in its ability to draw replicable and valid inferences from observed conversational data and its context (Krippendorff, 1980). While quantifying verbal data allows one to identify the frequency of actions, analyzing the same data qualitatively is an effective method to describe phenomena by creating or applying categories (Elo & Kyngäs, 2008; Mayring, 2004). The two content analyses in combination, offered implications regarding how students went through different (meta-)cognitive activities in terms of their quantity (i.e., frequency) and quality (i.e., types) and what made the differences during the problem-solving phase. Moreover, the categorization provided supplementary explanations of the results of the quantitative analysis.

To collect conversational data for the analyses explicated above, this study employed a PS-I method for learning about friction and work-kinetic energy within the physics subject area with sixteen recruited participants. The detailed descriptions of data collection methods and analysis are presented in the following subsections.

Material and participants

With the publisher's permission, this study used the problem scenario from Kapur's (2008, see pp. 419–421) work, which is carefully designed to be ill-structured. The problem scenario encompasses investigating a speeding case. The first author translated the problem scenario into Korean with the help of a bilingual American English professor. After translating the problem, the content validity of the material was reviewed and three possible solution pathways and associated learning concepts were identified by a high-school Physics teacher: uniformly accelerated motion, friction, and work-kinetic energy. During the review by the subject-matter expert, it was noted that because the problem scenario was inherently ill-structured, each solution path could be understood as conceptually on the right track but did not necessarily offer an error-free solution. Also, typical misconceptions for each target concept were also identified (see Table 3). In alignment with the identified target concepts, associated solution paths, and typical misconceptions, a 30-min-long instructional video, consisting of four parts, was developed by the authors with the help of the teacher. The first part offered an overview of the target concepts. Then, each solution path, its constraints and limitations, and typical misconceptions were addressed in the following parts. Table 3 summarizes the three target concepts that are relevant to the problem, the subcomponents of each major concept, and the parts of the problem scenario related to the concepts.

Kapur's (2008) problem scenario was originally designed for high school students. However, the authors assert that it was challenging enough for all participants in this study to experience failure because none of the teams managed to solve the problem. At the same time, its difficulty was appropriate to their level since every participant had to pass the preliminary test (more than 7 correct answers out of 10 questions), developed by the authors to assess their prerequisite knowledge, to participate in the study. The preliminary test assessed to what extent students understand the concepts of velocity, acceleration, the Newton's laws of motion (e.g., "Acceleration depends on two factors: (blank) and (blank): Fill out the blanks", "What is the formula for force?", and "If an electric scooter accelerates suddenly at 10 m/s^2, how much force was applied to the scooter?"). Sixteen students majoring in Educational Technology were recruited from a large private university in South Korea (thirteen women and three men; average age = 27; see Table 4). They received gift cards with a $5 value as a reward for their participation. The participants were randomly sorted into eight pairs.

Procedures and data collection

The study followed the conventional procedures of the PS-I approach (i.e., peer problem-solving followed by instruction; see Table 5). The participants worked with iPad Pro 2s and Apple Pencils during the study session and the default 'screen recording mode' was used to collect verbal and written data. For the problem-solving phase, the pairs of students were given 25 min to collaboratively solve the problem scenario, sharing one iPad and one Apple Pencil (see Fig. 1a). Each pair was provided with role scripts suggested by Westermann and Rummel (2012). The role scripts required students to alternate between 'questioner' and 'thinker' roles. The thinker verbalized their thought process, encouraging elaboration. The questioner listened and asked questions for clarification or correction, fostering metacognitive monitoring and questioning. They were also asked to generate multiple representations and solution methods as to the solution generation effect suggested by Kapur (2016). After the problem-solving phase, each student received an iPad and Apple Pencil and individually watched an instructional video. The participants were asked to watch the instructional video on the left-hand side of the screen with the annotated version of the worksheet on the right-hand side; they could take notes over their previous notes if necessary (see Fig. 1b).

After all the participants finished, the researchers selected two teams for individual interviews of 20 min each. The interviews were conducted to supplement the quantitative and qualitative analysis of the conversational data. The selections were made based on the authors' review of the conversation data, which showed students' impasses and struggles with the problem-solving activity. For the interview, the authors prepared initial

Table 3 Target concepts, subcomponents and related parts of the problem scenario reviewed and validated by a subject-matter expert

Target concept	Subcomponents	Examples of related components from Kapur's (2008) problem scenario	Typical misconceptions and errors
Uniformly accelerated motion (UAM)	- Distance - Speed - Time - The DST equations - Average velocity - The UAM equations	- From traffic police incident report, "speed limit on the road: 55km/h" and "evidence of skid marks: about 15m" - From eye-witness' account, "I think the boy took about 3 s to cross the road" - From medical examination report, "reaction time = 0.8 s on an average"	- Students often did not convert [km/h] into [m/s] - Students often paid attention to irrelevant values such as the width of the road
Friction	- Mass - Gravity - Normal force - Principle of action and reaction - The coefficient of friction - Frictional force equation	- From medical examination report, "weight = 75kg" - From mechanic, "this is a heavy car weighing about 1570kg" and "the coefficient of friction between the car's tires and the road is usually between 0.6 and 0.7"	- Students often confused the value of friction and the coefficient of friction - Students often confused the value of friction and the force equation - Students often ignored the normal force
Work-kinetic energy	- Work and energy - Formula for work - Kinetic energy equations - Amount of change in kinetic energy - Work and energy		- Students often failed to understand that work done by friction is the frictional force - Students often failed to associate the work with the change in kinetic energy

Table 4 Demographics of participants

No	Nickname	Group	Gender	Age	Interview
1	Sara	1	Female	28	X
2	Josh	1	Male	30	X
3	Kyu	2	Female	20	O
4	Min	2	Female	21	O
5	Eugine	3	Female	24	X
6	Grey	3	Female	30	X
7	So	4	Female	37	X
8	Kim	4	Female	29	X
9	John	5	Male	27	X
10	Jin	5	Female	24	X
11	Rachel	6	Female	26	X
12	Nartia	6	Male	39	X
13	Lang	7	Female	24	O
14	YJ	7	Female	22	O
15	Lucy	8	Female	24	X
16	Jane	8	Female	30	X

open-ended questions, avoiding explicit questions about student awareness of knowledge gaps. Instead, they were asked to describe their problem-solving processes, the moments they faced an impasse or experienced uncertainty, their actions or feelings at those moments and their immediate impressions at the end of the problem-solving phase. Additional follow-on questions were also asked to bring their underlying mental processes to the surface, particularly when the interviewers noticed the indicators that the students had (or had not) represented or reflected upon their activated knowledge (e.g., "could you try to remember the thoughts or feelings that you had at that moment and elaborate on them in your words?"). The verbal data in the recordings and interviews was subsequently transcribed.

Data analyses

This study conducted both quantitative and qualitative content analyses of the same conversational discourse data: deductive content analysis to examine the (meta-)cognitive actions that the learners took, followed by inductive content analysis to categorize students' judgments of their knowledge structure. Individual interviews held with four students were also analyzed as a means of triangulating the results of the content analyses. Figure 2 shows the overall procedure of data analyses.

Deductive content analysis of the problem-solving discourse

Regarding the first research question, the authors created a coding scheme to deductively examine learners' conversational data during the problem-solving phase, by adapting coding schemes from previous research (summarized in Table 2 above). In drafting the scheme, the authors first categorized three phases of problem-solving (i.e., problem analysis and representation, solution development, and solution evaluation). Then, seven

Table 5 PS-I implementation procedure

Week	Process	Time	Task	Data
1st week	Participant selection	20 min	Preliminary test	- Test results for participant selection
2nd week	Problem-solving	25 min	Collaborative problem-solving	- Recorded conversation - Representation and solution methods generated during problem-solving - Researchers' filed notes during observation
	Instruction	60 min	Individual learning with instructional video	- Students/ notes during instruction - Researchers' filed notes during observation
	Interview	20 min per person	Interviews with two selected pairs	- Recorded interview

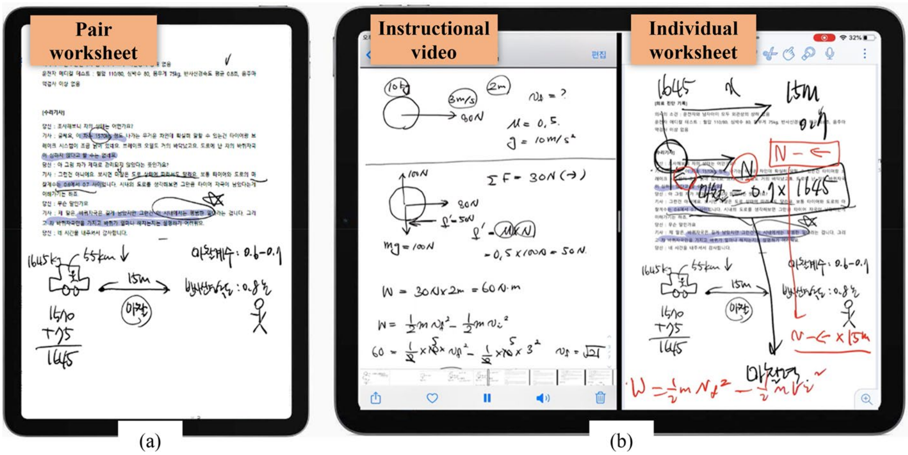

Fig. 1 Sara and Josh's pair worksheet (**a**) and Sara's individual worksheet during the instruction phase (**b**)

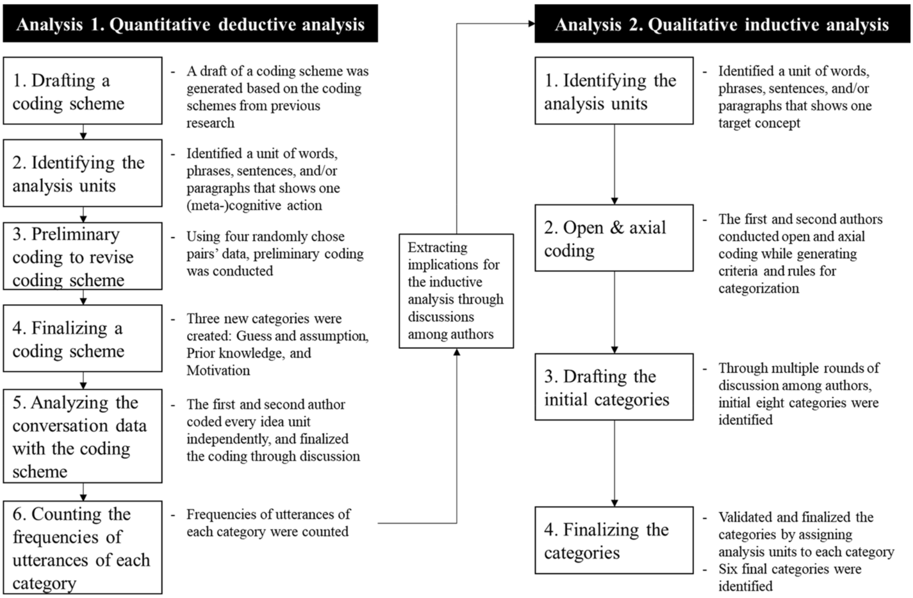

Fig. 2 The overall procedure of data analyses

preliminary codes for cognitive and metacognitive actions for each phase were generated: *problem assessment, impasse, solution development, recognition of error, identification of reasons for the error*, *solution evaluation without error detection* and *orientation*. In the process, authors adapted some codes from Kapur and Kinzer (2009), including *problem assessment, solution development, solution evaluation, and orientation*, and other codes from Große and Renkl (2007), such as *impasse, recognition of error,* and *identification of reasons for the error*. A combination of codes from these two studies was adopted because

Table 6 Coding scheme for cognitive and metacognitive behaviors across problem-solving phases

Phases	Codes	Description	Example	Action type
Problem analysis and representation	Problem assessment (PA)	Mentions details of a problem, prioritizes details, makes connections between details, or provides new perspectives in interpreting aspects of a problem	"Um… For now, I think what counts is… the total weight and time, not the total weight and the speed limit. like we thought."	Cognitive
Solution development	Solution development (SD)	Mentions a (sub-)goal, an operator, or a relevant formula, tries anticipative reasoning, and/or suggests ideas, proposals, alternatives, or rebuttals	"Distance 15, velocity 55. Shall we calculate the speed using distance velocity and time (DVT)?"	Cognitive
	Guess and assumption (GA)	Guesses next solution steps, and/or makes assumptions on concepts and procedures to assess a problem and generate and evaluate a solution	"But since high coefficient of friction means a stronger friction, doesn't it mean the brake should work faster? That's how I see it."	Cognitive
Solution evaluation	Solution evaluation without error detection (SE)	Gives an evaluation of the generated solution and/or reflection on their problem-solving processes (but does not find any potential errors)	"When we say this is 25, what's the reason for dividing it with the weight? … I mean we divided the total distance… right? I think what we did is right!"	Cognitive and/or metacognitive
	Recognition of error (RE)	Mentions that there is an error, finds, and/or reflect upon an error in the generated solution	"This is 4.5 again? Maybe not, I don't think this is it, obviously, 559."	Cognitive and/or metacognitive
	Identification of reasons for the error (IRE)	Gives reasons for the errors after reflections on their problem-solving processes	"I think the coefficient of friction we set here should not be 1."	Cognitive and/or metacognitive

Table 6 (continued)

Phases	Codes	Description	Example	Action type
Phase-independent	Impasse (IM)	Expresses confusion and uncertainty, pauses for a long period, and/or states unexpected difficulty	"Coefficient of friction… apply the coefficient of friction. Then the way is… (long pause)"	Cognitive
	Prior knowledge (PK)	Activates prior knowledge and/or aligns it with new information	"Suppose the distance is 15 m, then, um, this is X and it would be best to know the acceleration inclination, mmm F = ma…"	Cognitive
	Motivation (M)	States motivation or plans to learn need-to-know knowledge from the following instruction	"I think there was something like square of the weight. 70… no 1675. I hope the video shows this part."	Metacognitive
	Orientation (O)	Tries to plan, monitor, orient and/or guide the group's process	"Let's write down and then think about it"	Metacognitive

the codes from the former correspond well to the three phases of problem-solving commonly identified in previous research, while the latter specifies (meta-)cognitive actions of recognizing and reacting to knowledge gaps which are of interest in this study. Also, it should be noted that the codes *impasse* and *orientation* represent a phase-independent action, while others can be found in one of the three phases (see Table 6). Subsequently, as preparation for the deductive analysis, the authors divided the conversation transcripts into units of analysis made up of combinations of words, phrases, sentences, and paragraphs. The determination of units was made on the basis that that a single unit only included one (meta-)cognitive action. For example, the utterances, "so, 55km/h is… so when you drive faster than that… umm… I can't do this because I don't know the formula for this, I forgot" was divided into two units as it represented one cognitive action (representing the problem element of speed limit) and one meta-cognitive action (recognizing impasse). The authors then conducted preliminary coding using the initial draft of coding scheme with data from four randomly chosen pairs. During the preliminary coding process, three new codes were created: *Guess and Assumption, Prior Knowledge* and *Motivation*. After closely reading the transcripts, it became apparent that the learners frequently made assumptions based on the context of the problem, such as "I think the coefficient of friction has something to do with the weight of the car". The authors created a new code for this sort of utterance, *Guess and Assumption*, because of the belief that making assumptions can be a starting point for AKG. Moreover, *Prior Knowledge* was added to the codes because the learners often retrieved relevant concepts when assessing the problem or generating/evaluating solutions. Aligning new information with prior knowledge can be a sign of learners revisiting their memory and preparing to generate a solution, which often leads to global AKG (Loibl & Rummel, 2014). Furthermore, after learners realized that they needed more knowledge to solve the problem, some of them stated that they hoped to learn this from the following instruction. The authors coded these utterances as *Motivation*. It should be noted that the codes *prior knowledge* and *motivation* were also categorized as phase-independent actions.

The first and second authors then coded every idea unit based on the final coding scheme (see Table 6). The inter-rater reliability was satisfactory (Cohen's $\kappa = 0.74$) and the final codes were determined by resolving discrepancies through discussion.

Inductive content analysis for the categorization of students' judgements of knowledge structure

An inductive qualitative content analysis was conducted to answer the second research question after drawing on implications from the first content analysis. The implications were drawn through the discussion of the newly identified codes and the quantitative results from the first analysis. First, while applying the deductive approach, new categories of problem-solving behavior emerged: attempts to guess the next step of the solution, prior knowledge alignment and motivation (i.e., Guess and assumptions, Prior knowledge and Motivation). The authors identified that these utterances (except Prior knowledge) were presented by *some* but *not all* participants. Moreover, the lopsided distribution of behavior frequencies was particularly noticeable in the categories involving metacognitive actions (e.g., Self-evaluation, Recognition of errors, Identification of reasons for the error, and Orientation; see Table 7 in the Findings section). Based on this finding, the authors conducted the inductive content analysis to explore how the learners' knowledge structure varied with their problem-solving behaviors. The authors assumed that students' metacognitive actions affected their judgment of knowledge structure and, accordingly, led to different experiences of AKG.

A new unit of analysis was selected because the aim of the second content analysis was different from that of the first analysis: to analyze how learners judge their current knowledge structure and explore the variations in learners' AKG experiences. Knowledge structure in an individual learner vary among different target concepts. Thus, idea units were determined based on phrases that signaled a target concept (i.e., uniformly accelerated motion, friction, or work-kinetic energy as indicted in Table 3) For example,

> 133. A: "Well, the frictional force gets bigger when the weight increases, right? And the larger the force is… Nope? Another way we should dig into?"
> 134. B: "Hmm… 2.2m…."
> (….)
> 138. A: "I don't think he speeded".
> 139. B: "Because the kid suddenly ran into the road, and that's why there was a 15m-long skid mark. The kid passed for 2.2 seconds meaning that the car moved 15m for 2.2 seconds. Then… 15m for 2.2 seconds…"
> 140. A: "Well, 55km/h means that it drives 55km per hour…".

These multiple turns of conversation were divided into two units of analysis: the first part (line 133) where A was addressing his prior knowledge of *frictional force* and the second part (lines 134–140) where A and B were discussing a problem about the concept of *uniformly accelerated motion*. This level of analysis units allowed authors to consider a knowledge structure of a target concept as a unit of AKG. That is, AKG can be captured as a micro unit of each knowledge element, rather than as a whole impression of a challenging problem-solving experience.

After identifying all of the idea units, the first and second authors subsequently conducted open and axial coding. During the process, criteria and rules were devised for the categorization of students' judgement of knowledge structure. The categories were based on:

A. *whether* learners had *represented* the knowledge in their problem space and if so,
B. *whether* they had *reflected on* the represented knowledge and if so,
C. *how* they judged their knowledge structure based on this reflection.

Each rule was implemented only when the preceding rule was satisfied. For example, if learners had not represented a certain knowledge element, there could be no reflection or judgement about the knowledge element; rules B and C were not applied in determining which category of knowledge structure learners judged as regarding the corresponding knowledge element. Likewise, when learners had not reflected on the represented knowledge, rule C was not applied because there could be no judgement of knowledge structure without any reflection.

Through multiple rounds of discussion among the authors, a total of eight categories were initially identified: *Knowledge not Represented; Knowledge not Reflected;* Knowledge judged as *Sufficient; Intuitive; Naïve; Uncertain*; *Insufficient with Unspecified Knowledge Gaps*; *Insufficient with Specified Knowledge Gaps*. To validate and finalize the categories, the coders then assigned each analysis unit to one of these categories. During the process, the coders only considered explicit *verbal* data from the recorded conversations. In the discourse, explicit signs of reflection or self-evaluation (e.g., 'I think we were wrong here')

Table 7 Descriptive statistics of problem-solving behaviors

	PA	SD	GA	SE	RE	IRE	IM	PK	M	O	Total
G1	94 (61.4%)	13 (8.5%)	3 (2.0%)	0 (0%)	0 (0%)	0 (0%)	10 (6.5%)	14 (9.2%)	3 (2.0%)	16 (10.5%)	153 (100%)
G2	31 (32.0%)	3 (3.1%)	17 (17.5%)	5 (5.2%)	8 (8.2%)	1 (1.0%)	11 (11.3%)	10 (10.3%)	0 (0%)	11 (11.3%)	97 (100%)
G3	22 (34.4%)	9 (14.1%)	4 (6.3%)	0 (0%)	4 (6.3%)	3 (4.7%)	9 (14.1%)	2 (3.1%)	2 (3.1%)	9 (14.1%)	64 (100%)
G4	32 (32.7%)	4 (4.1%)	11 (11.2%)	0 (0%)	5 (5.1%)	3 (3.1%)	19 (19.4%)	12 (12.2%)	2 (2.0%)	10 (10.2%)	98 (100%)
G5	30 (39.0%)	2 (2.6%)	13 (16.9%)	1 (1.3%)	1 (1.3%)	0 (0%)	12 (15.6%)	1 (1.3%)	0 (0%)	17 (22.1%)	77 (100%)
G6	28 (40.0%)	0 (0%)	23 (32.9%)	3 (4.3%)	0 (0%)	0 (0%)	3 (4.3%)	2 (2.9%)	0 (0%)	11 (15.7%)	70 (100%)
G7	38 (51.4%)	7 (9.5%)	0 (0%)	1 (1.4%)	1 (1.4%)	1 (1.4%)	18 (24.3%)	2 (2.7%)	3 (4.1%)	3 (4.1%)	74 (100%)
G8	35 (41.2%)	13 (15.3%)	3 (3.5%)	10 (11.8%)	4 (4.7%)	2 (2.4%)	8 (9.4%)	1 (1.2%)	0 (0%)	9 (10.6%)	85 (100%)
Total	310 (43.2%)	51 (7.1%)	74 (10.3%)	20 (2.8%)	23 (3.2%)	10 (1.4%)	90 (12.5%)	44 (6.1%)	10 (1.4%)	86 (12.0%)	718 (100%)

PA Problem assessment, *SD* Solution development, *GA* Guess and assumption, *SE* Solution evaluation without error detection, *RE* Recognition of error, *IRE* Identification of reasons for the error, *IM* Impasse, *PK* Prior knowledge, *M* Motivation, *O* Orientation were rarely presented, although some remarks (e.g., 'If I know the formula to use this…') could be inferred to show reflection. The coders used these signs as criteria for reflection. Occasionally, there were non-verbal signs that indicated that students seemed to reflect on their knowledge, such as a long pause after they finished a sub-step in solution generation process. However, these signs were not considered because no other direct evidence that explicitly showed students' reflection.

In the process of finalizing the categories, the *intuitive* category was removed because intuitive knowledge was always either *sufficient, naïve* or *uncertain* (e.g., "This reminds me of energy because everything has energy, but I don't know whether we are on the right track. Do you?") but not vice versa. Similarly, the coders agreed to remove *uncertain* because they believed that uncertainty was omnipresent when students self-evaluated their knowledge as *naïve* or *insufficient*.

In addition, semi-structured interview data from four students was reviewed to consider unrepresented (meta)cognitive processes for the finalization of the categories. Lastly, authors identified the cognitive and metacognitive processes that result in each category of students' judgement of knowledge structures. The detailed findings from the inductive analysis of the conversations and interviews are provided in the following sections.

Findings

The findings in this study are presented in two parts. Part I responds to the first research question and comprises a quantitative description of the results of the deductive content analysis with the adapted coding scheme. Part II, which addresses the second question, presents the final categorization of the different judgments of knowledge structures and (meta-)cognitive actions that yield each category of judgment.

Part I: Cognitive and metacognitive actions during the problem-solving

As shown in Table 7, students spend most of their time analyzing the problem situation (e.g., Problem assessment) or struggling to solve the problem (e.g., Impasse, Solution development, Guess and assumption, Prior knowledge, Orientation) but seldom evaluated their ideas and reasoning (e.g., Self-evaluation, Recognition of errors, Identification of reasons for the error) in the problem-solving phase. Over 43% of utterances, out of a total of 718 idea units, were assigned to problem assessment. This was consistent across all eight groups. In general, students seldom engaged in reflection, such as recognizing an error (utterance count = 23), identifying the reasons for an error (utterance count = 10) or other types of solution evaluation (utterance count = 20). However, they frequently faced moments of impasse (utterance count = 90), made assumptions about concepts or processes (utterance count = 74) and retrieved prior knowledge (utterance count = 44).

The problem given to students was ill-structured and inherently entailed uncertain and ill-defined situational elements; it is accordingly assumed to make students confused in their representation and interpretation. Given the nature of the problem, it is not surprising that 72.8% of the problem-solving behavior was affiliated with Problem assessment, Impasse and Guess and assumption. Even though investigating the sequential relationships between problem-solving behaviors is beyond the scope of this research, it is not a logical leap that students faced an impasse in analyzing the problem and thus put most of their efforts into representing the problem situation.

It can be also assumed that the students produced many hypotheses and assumptions because of their uncertainty about the problem parameters. Two participants who were interviewed, Kyu and Min, reported that they retrieved some seemingly relevant knowledge after facing a moment of impasse and tried to make sense of the parameters that confused them. It is thus assumed that they were struggling to represent the given problem and decided to reduce the apparent discrepancies (i.e., means-ends analysis) rather than collectively assess the constraints or possible solution paths. This is not to say that the means-ends analysis strategy does not allow them to explore constraints and possible solution paths. Rather, it can be an effective way for some students to navigate the problem space, grasp the essence of the problem, and generate solutions (Smith et al., 1994). However, for other students, like Kyu and Min, the means-ends analysis strategy may limit opportunities for exploring the problem space more extensively or deeply. This is especially the case when students become cognitively fixated on eliminating apparent gaps they identify (Sweller, 1988; Youssef et al., 2012). This is demonstrated by both their dialogue transcript (see Table 8) and their description in the interview of what happened in the moment.

Kyu and Min initially wrestled with the coefficient of friction to assess whether the car was speeding (Turns 1–3). However, as they noticed that there were multiple factors they should consider (Turns 4–6), they stopped exploring the problem space they deemed relevant to the friction. Instead, they began focusing on using the formula of distance, velocity, and time (DVT) as they believed it would help them address the gaps they identified and arrive at a plausible answer (Turns 7–9). During her interview, Min described their problem-solving process, explaining how they grappled with various pieces of information related to friction that they ultimately couldn't fully grasp. As a result, they resorted to applying the distance, velocity, and time formula based on the information they were able to comprehend.

Min: 'So we read the problems and drew a picture to make understanding. Since the car weighs a lot, it would obviously be related to friction, and since it takes 3 s to cross the road, that's also related to friction. But the problem mentions the body weight and vehicle mass, and we also have to consider the reaction time so we were at a loss. So just doing what we can, we remembered the distance time and speed equation and put in the given information regarding distance, time and speed and tried to get an answer that looked reasonable'.

Meanwhile, it is worth noting the lopsided distribution of the behaviors that involved evaluating the group's solutions or ideas (i.e., Self-evaluation, Recognition of errors and Identification of reasons for the error). Despite the overall low frequency of such behaviors, the authors observed notable variation among the groups; some groups never discussed these topics or did so in less than five percent of their utterances; for other groups, these categories comprised over ten percent of their total utterances.

The authors presumed that these variations impacted students' judgement of knowledge structure, as AKG must follow reflection. Thus, the second (qualitative, inductive) content analysis was conducted to categorize the different judgement of knowledge structures and determine the patterns and processes by which students do or do not become aware of their knowledge gaps. The findings of the second content analysis are presented in the following section.

Table 8 Excerpt illustrating how Min and Kyu approached the problem after facing an impasse

Turn	Person	Excerpt
1	Min	Wait, the coefficient of friction is from 0.6 to 0.7… Are we to get the average? 0.65?
2	Kyu	Hey, or get the maximum and minimum value. From the minimum of 25kmph… divide it by 0.7? Um, but… It's similar to the number we got before
3	Min	Hmmm…. Is that so…
4	Kyu	Hey, then let's just put down all the given information. (Draws a picture of the situation) (omitted)
5	Min	We should consider the reaction time… ah, but the weight and friction seem to be related together…
6	Kyu	Really? Then let's put in the coefficient of friction…
7	Min	No… Maybe we're supposed to use the 'distance speed time formula'?
8	Kyu	Maybe, shall we multiply speed and time?
9	Min	Yeah, I feel like we then can get an answer. Let's go for it
10	Kyu	Ok, let's do it. Multiplication is my thing

Part II: How students judge their knowledge structures

The findings of the second content analysis are presented in this part. A total of six categories were identified from the analysis. The examples of student conversation during problem-solving that are classified in each category and the excerpts from the interview that elaborated the (meta-)cognitive actions of each category are presented. The last section of this part summarizes the six categories, their definitions, and the (meta-)cognitive actions that distinguish them.

Category a. Students do not represent the target knowledge

The first category is *knowledge not represented,* where students do not represent the target knowledge (See Table 3 for the target knowledge of the problem). This category is not related to the experience of AKG; students do not recognize any knowledge gaps between their knowledge and the target knowledge which they have never represented. An excerpt from the conversation between Lucy and Jane in Table 9 illustrates how the two students ended up not representing the target knowledge of uniformly accelerated motion.

In solving the problem, Lucy and Jane decided to take an argumentative approach rather than a mathematical or scientific one after they realized they were unable to solve the given problem (Turn 4). That is, they failed to position parameters such as 'time', 'velocity' and 'distance' within their problem space and focused on generating alternative solution paths by examining other contextual information. After they found a possible alternative solution, which was irrelevant to the learning concepts, they finished their problem-solving without exploring other solution paths. As a result, they seemed to have a global awareness of a knowledge gap for some knowledge elements (e.g., the mathematical relationship between distance, velocity and time) but failed to recognize the actual gap between their current knowledge and the target knowledge. That is, a supposed precondition of AKG did not occur: representation of the target knowledge.

Table 9 Excerpt illustrating how Lucy and Jane ended up following an irrelevant solution path

Turn	Person	Excerpt
1	Lucy	Do you think this 3 s plays an important role in the problem?
2	Jane	3 s…
3	Lucy	The 3 s here, and also in the latter part we have a lot of numbers, like how many kms or distance…? Are we supposed to approach these numbers mathematically?
4	Jane	Mathematically… But we don't know how they're related. So just going with the guts, 3 s is a very short time, can it be an evidence that this driver stopped at the sight of the boy? I mean this person did what he could? An inevitable accident?
5	Lucy	Ah, just speeding and not hitting the child?
6	Jane	I mean this person didn't speed to go forward, but perhaps to avoid and stop?
		(Omitted)
7	Jane	I keep rooting for speeding. Ultimately he was fined for speeding in this case. To pay the penalty or to plead not guilty… So as evidence, um, firstly he was already speeding, over 55kmph, which seems like the biggest part, and secondly, he already has history of speeding?
8	Lucy	But that might offend the client. Um, these may explain the current situation, but it seems to lack evidence for speeding
9	Jane	Yes, I thought so too, because in an opposite view, same accident can happen even if he was driving for 40kmph
10	Lucy	Any other grounds? His driving experience is 34 years, and he was penalized only twice. Still, violation is a violation, right?
11	Jane	So he's not that bad of a person…?

Table 10 Excerpt illustrating how Grey and Eugine addressed the represented knowledge of friction

Turn	Person	Excerpt
1	Grey	Coefficient of friction? So we thought the car skidded much, but it can differ according to the road
2	Eugine	Yeah, the weight of the car too…
3	Grey	A heavy vehicle, thus it can't help but skid more…
4	Eugine	The heavier the higher the friction, so won't it brake shorter?
5	Grey	You think so?
6	Eugine	So what problem do we need to solve here…
	Grey	Whether to defend or persuade payment. So does that mean a heavier car can stop faster?
7	Eugine	No. It stops slower
8	Grey	But since high coefficient of friction means a stronger friction, doesn't it mean the brake should work faster? That's how I see it
9	Eugine	Or in my opinion, because it's heavier, I thought the inertia would be stronger and it might have caused longer time to stop
10	Grey	I see. If you look here, the mechanic says the skid marks on the road is bad…
11	Eugine	What bothers me is this. If it's an ambassador car, he must work there… (omitted) In the witness' statement the boy took about 3 s to cross the road. He doesn't know if the boy looked at the traffic light, the boy just seemed to follow the ball. Nevertheless, the driver braked and the boy wasn't injured

Category b. Students do not reflect on their current knowledge

The second category is *knowledge not reflected,* where students represent the target knowledge but do not reflect on their current knowledge. By reflection or reflecting on, we refer to a process of consciously thinking about one's current understanding and knowledge structure. This category is also not related to the experience of AKG; students do not recognize any knowledge gaps regarding the knowledge that is represented but not reflected on. An excerpt from the conversation between Grey and Eugine in Table 10 shows that they addressed the knowledge of friction, but did not reflect on it.

In their process of generating solutions, Grey and Eugine made assumptions based on their understanding of the relationships between the coefficient of friction, frictional force and weight. They then discussed whether the client was speeding. During the discussion, they brought other scientific concepts (e.g., inertia) and parameters (e.g., skid marks and time) into their problem space. In the process, they engaged in a discussion regarding the definition and features of the coefficient of friction (Turns 1–7), but they did not reflect on whether their understanding of the coefficient of friction was appropriate or adequate for solving the problem. Grey and Eugine did not circle back to the concept of friction, resulting in a missed opportunity to experience AKG through reflection of the concept.

Category c-1. Students judge their current knowledge as sufficient

The third category is *sufficient,* where students represent and reflect on the target knowledge, and judge their current knowledge structure as sufficient. This category still does not involve the experience of AKG; students do not recognize the knowledge gaps in their knowledge structure, rather, they inaccurately judge their knowledge structure as sufficient

Table 11 Excerpt illustrating how Grey and Eugine reflected on their activated knowledge and regarded it as sufficient

Turn	Person	Excerpt
1	Eugine	Distance divided by time. I mean, the speed…
2	Grey	When we say this is 25, what's the reason for dividing it with the weight? Dividing it with the weight suggests, inversely, we used multiples, by multiplying the time the weight can travel, I mean we divided the total distance… right? I think what we did is right
3	Eugine	Looks about right?
4	Grey	Yeah, seems alright to me?
5	Eugine	Then we should write a report…
6	Grey	Yeah, since he went over the 55kmph speed limit… it's speeding. Just write it here

when it is in fact insufficient. An excerpt from the conversation between Grey and Eugine illustrates how they judge their knowledge about the equation of motion as sufficient for generating a correct solution, which is in fact incorrect.

The excerpt in Table 11 took place after Grey and Eugine discussed a possible solution path related to friction without reflecting on it (shown in Table 10 in the previous subsection *Category b.*). This time, Grey and Eugine self-evaluated their attempt to apply the equation of motion to figure out whether the client was speeding. They agreed that they had successfully solved the equation using the given parameters (i.e., time, distance, and weight) and that their judgment was correct (Turns 3–6). However, they were not aware that their self-monitoring had blind spots—their equation was wrong to calculate the initial speed of the car. They again failed to perceive the existing knowledge gap.

Category c-2. Students judge their current knowledge as naïve

The fourth category is *naïve,* where students represent and reflect on the target knowledge and judge their knowledge structure as naïve. From this fourth category, students begin to recognize their knowledge gaps. When judging a knowledge structure as naïve, they recognize the existence of potential knowledge gaps (i.e., global awareness of knowledge gaps). However, with this judgement, students are not certain whether their knowledge structure is sufficient or insufficient because they are not aware of any criteria to evaluate whether they are right or wrong. An excerpt from the interview with Kyu and Min illustrates how they reflected on the represented knowledge of uniformly accelerated motion and considered that their knowledge structure was naïve.

As shown in Table 12, when Kyu and Min were applying the equation of "distance, speed, and time formula" to generate a solution (Turns 1–5), they came up with a number that they considered as a potentially correct answer simply because it was close to the parameter given in the problem (Turns 6–7). Their reflection lacked an external standard for comparison and failed to motivate them to explore further. That is, while they were not confident in their alternative solution, they had no explicit or implicit sense of whether they were wrong.

Table 12 Excerpt illustrating how Kyu and Min reflected on their activated knowledge and regarded it as naïve

Turn	Person	Excerpt
1	Kyu	Yeah. Shall I multiply by speed? To 0.8?
2	Min	Yes, I think we should, because…
3	Kyu	We can't have a thousand digit
4	Min	And 15 m…
5	Kyu	Ah, we should use 2.2 s, subtracting 0.8 from 3 s
		(Omitted)
6	Min	62.5… Huh? I think we got it… what do you think?
7	Kyu	Yeah, a little. But I think we were lucky…. Um…

Min: "As we said before, we had no idea where to begin with at first. Then I remembered distance speed time formula, so we put the numbers in accordingly. (…) The numbers seemed about right. But what's the right answer?".

In fact, Min never recognized the errors in her group's final solution and hoped that it was correct. She had a global sense that there might be a gap between her knowledge and the learning concept. However, she did not feel a need to further explore or reflect on the knowledge elements because she thought it was possible that there was no knowledge gap (i.e., their answer could be right).

Category c-3. Students judge their current knowledge as insufficient without specifying the knowledge gaps

The fifth category is *insufficient and unspecified knowledge gaps,* where students represent and reflect on the target knowledge, and then recognize the insufficiency of their knowledge to generate a correct solution. When judging their knowledge structure as insufficient, students know that their knowledge is not complete or lacking important knowledge elements. However, they do not specify what is missing or incoherent—their awareness of knowledge gaps is global rather than specific. An excerpt from the interview with Kyu and Min illustrates how they reflected on the represented knowledge of friction and regarded their knowledge as insufficient. Kyu and Min tried to make sense of the coefficient of friction in calculating the car's initial speed (Turns 1–7). However, they rapidly faced an impasse and realized that they did not know the relevant concepts and procedures to generate a solution (Turns 8–10).

Kyu and Min were trying to make sense of what the coefficient of friction is and how they could apply this notion in calculating the car's initial speed (Turns 1–7). However, they shortly faced an impasse and realized that they did not know the relevant concepts and procedures to generate a solution (Turns 8–10). As Kyu and Min did initially (see Table 13), some students realize that their approach or application of knowledge is insufficient or inappropriate for the problem but fail to identify why. They have a global sense that their process is incomplete but they do not know which part of their knowledge structure is lacking. Even though students may find errors or inconsistencies in their solutions, they rarely figure out why the errors occurred. In this case, some students give up further exploration and look for another approach.

Kyu: "Before coming to 'distance speed and time formula', what is it, we substituted the values… ah, the traction, we went with our guts and substituted the values, but all

Table 13 Excerpt illustrating how Kyu and Min recognized the existence of a knowledge gap

Turn	Person	Excerpt
1	Kyu	The car stopped. So… how do I say…. He put on the brake, therefore friction occurs, and speed…
2	Min	Coefficient of friction sounds like… a number between 0 and 1…
3	Kyu	Right? Perhaps…
4	Min	If we multiply distance by coefficient of friction…
5	Kyu	Distance?
6	Min	My bad, multiply speed by coefficient of friction, distance…
7	Kyu	Then x is the speed?
8	Min	Won't it? Ah, but it totally doesn't sound right. I don't know what's what
9	Kyu	Then if we multiply by 15…. 9… is that right?
10	Min	Wait, no…. (Omitted) Wow, this is hard

the answers didn't look right. We did get a value at some point, but I thought we were just scribbling, so I was saying 'no, this is not right'…. I didn't know what to do with it. I thought it was too difficult, so that's why I went for the 'distance speed and time formula."

Category c-4. Students judge their current knowledge as insufficient and specify the knowledge gaps

The sixth category is *insufficient and specified knowledge gaps,* where students represent and reflect on the target knowledge, and recognize and specify what is insufficient in their knowledge structure. In this category, students specify what they know and what they do not know (i.e., specified knowledge gaps). An excerpt from the interview with Lang and YJ illustrates how they reflect on their knowledge structure and specify the gaps they need to fill in order to generate a complete solution.

As shown in Table 14, Lang and YJ had difficulties mapping the given parameters (e.g., width of the road, distance, speed, and others) onto their problem space. They then recognized that their knowledge was insufficient to make a plausible solution (Turn 3) and continued struggling to represent the problem and looking for elements they had overlooked (Turns 5, 9, and 12). Ultimately, they realized that they needed a formula related to the given parameters, took another approach and generated a solution. However, unlike the problem-solving behaviors in the other five categories, in this case, the students came back to their initial solution path to reflect after they had come up with a solution. This action is evidenced in the interviews with Lang and YJ.

Lang: "At first we really made an effort to get the speed. But realizing that if we don't know the formula, we can't solve the problem, I suggested approaching it logically, so we searched for other evidences. But it doing so, it was all circumstantial evidences so we couldn't make a legal judgment. And since we have these numbers anyway, we felt the need for a specific evidence, so we went back to see if we misunderstood or missed any other points."

YJ: "Well, in solving the problem I wasn't sure what the coefficient of friction was. Whether it was a number between 0 and 1, or just the number 0.6 was enough, and how to put the numbers in what equation… we only made guesses. We wished earnestly that at

Table 14 Excerpt illustrating how Lang and YJ specified their knowledge gap

Turn	Person	Excerpt
1	Lang	Depending on the width of the road… no, should I say the distance of the possible hitting point…
2	YJ	Ah, that's what we should calculate…
3	Lang	If you look at the PS, it's speeding when driving above the speed limit. But we can't calculate the speed now. We don't know the formula, what the coefficient of friction is, and all these numbers are ambiguous, so I don't know what to say…
4	YJ	Yeah…
5	Lang	Don't the cops get the speed with a speedometer?
6	YJ	Yes
		(Omitted)
7	YJ	Whatever. I wish at least I knew a formula to begin with…
8	Lang	Then let's do what we can do. Let's approach with logic. We can't calculate with 'distance speed and time'. Neither of us know how to
9	YJ	Okay. Here, the boy wasn't injured. Let's put a huge significance there…
10	Lang	But isn't the fact the car didn't hit the boy a minor issue? To prove speeding…
11	YJ	You have a point…
12	Lang	The police officer fined him Rs. 20,000 for speeding, so we know that he was speeding regardless of whether he hit the boy or not. The uninjured boy doesn't seem to have much to do with it. And he already has history of speeding and drunk driving. He's really bad. Looking at the history… don't you think the officer made a right judgment?
13	YJ	Then, shall we root for paying the fine?
		(Omitted)
14	Lang	This is what we have for now, but I want to use a formula to calculate…
15	YJ	The issue is we don't know how to…
16	Lang	Well the fact that 15 m skid mark is not something new would mean the skid mark ultimately is a meaningless information?
17	YJ	Is it absolutely meaningless?
18	Lang	Not sure…
19	YJ	I hope the explanation video covers this later

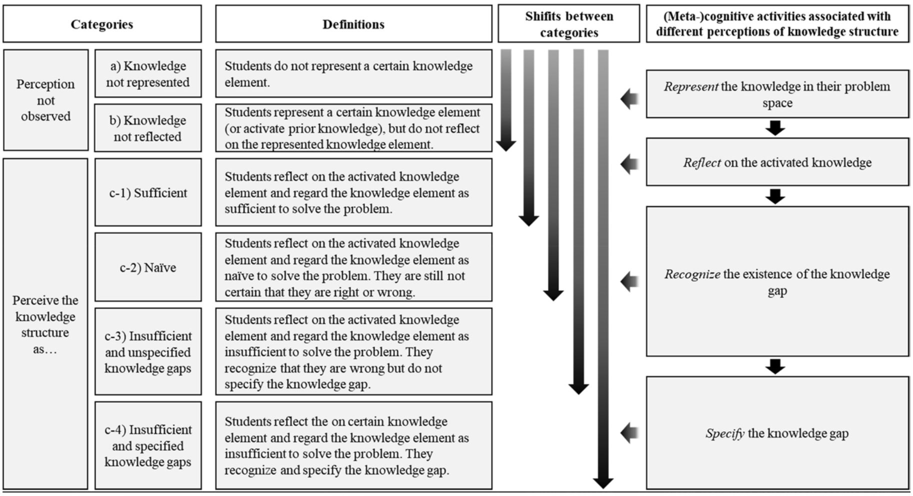

Fig. 3 Categories of students' judgements of knowledge structure

least we knew a formula to get the speed using the coefficient of friction. Then we could have tried putting in the numbers in different ways… it was a pity."

Summary of the second content analysis

The second content analysis categorized six different judgements of knowledge structures (see Fig. 3). As stated in the methodology section above, cognitive and metacognitive actions of the representation of knowledge, reflection on the represented knowledge and the results of this reflection determined the limits of each category. That is, the specific actions of *representing, reflecting, recognizing* and *specifying* were used to assign a student's judgement of knowledge structure to a particular category. Three categories were specifically related to different experiences of AKG: c-2, c-3 and c-4. The other three categories (a, b and c-1) indicate that students could not perceive any gap between their knowledge and the target knowledge.

Discussion

To understand students' experiences of AKG and its role in PS-I contexts, previous studies have explored the dimensions of specificity (i.e., global vs. specified; Loibl & Rummel, 2014) and the level (i.e., 6-point Likert-scale, Glogger-Frey et al., 2015; larger vs. smaller, Litman et al., 2005) of AKG. In addition, studies in cognitive psychology have revealed the particular issues that raise feelings of insufficiency or incoherence (e.g., procedural bugs, conceptual conflicts, and prediction failures; Chi, 2000; Gruber & Ranganath, 2019; VanLehn, 1999). Building on these studies, the current investigation aimed to strengthen the understanding of the cognitive and metacognitive processes that yield different experiences of AKG among individuals. With a particular focus on impasse and reflection processes, this study quantitatively investigated the relative frequencies of cognitive and metacognitive actions that took place during problem-solving. It then proceeded to qualitatively explore how students judge their knowledge structures and the (meta-)cognitive actions (e.g., representing, reflecting, recognizing, and specifying) that lead to different judgements.

The current study considered a knowledge structure of a target concept (e.g., uniformly accelerated motion, friction, or work-kinetic energy) as a unit of AKG rather than treating a whole impression of problem-solving experience as an AKG unit (e.g., "I struggled to generate a solution", "My knowledge was insufficient to complete the task"). This micro unit of AKG, rather than a "feeling-of-not-knowing" in a broader sense, has been also applied in previous PS-I studies, although not explicitly addressed, which emphasized the importance of highlighting the deep structure of the target concept. For example, presenting contrasting cases that differ in one knowledge element at a time may help students detect incoherence in their solutions by highlighting the deep features of the target concept (Loibl et al., 2017).

As Loibl et al. (2017) elaborated, the effectiveness of PS-I in knowledge acquisition may reach its full potential when (a) students activate relevant prior knowledge; (b) identify gaps between their current knowledge structure and that of the target knowledge; and (c) recognize the deep features of the target knowledge. Lacking an instructional design element that facilitates one or more of these three features may significantly reduce the efficacy of a PS-I approach. Unfortunately, some might misunderstand the experience of AKG in PS-I as having a broader "feeling-of-not-knowing" and fail to recognize the lack of specification of students' awareness of knowledge gaps. If so, the students would then be likely to have an unproductive learning experience, since they may have reduced chances of recognizing the deep features of the target concept. Therefore, designing learning activities with the understanding of AKG in a target knowledge element is important to reaping the benefits from a PS-I approach.

Meanwhile, the analysis of AKG experiences using the micro AKG units allowed us to identify students' different judgements of their knowledge structure, especially those of non-AKG experiences. In many cases, students failed to reflect on, or even represent, a knowledge element of a target concept. This finding is consistent with many studies in the problem-solving literature. During the problem-solving phase of PS-I, students often fail to construct an appropriate problem space (Kapur, 2016) or struggle to retrieve the relevant prior knowledge and thus fail to access the target knowledge. This failure to represent the problem appropriately is partially due to the ill-structured nature of the given problem (Ge & Land, 2003), as students are distracted by many of the apparently vital issues surrounding the problem. In addition, the lack of situational and domain knowledge also hinders novice problem-solvers from recognizing germane parameters (de Jong & Ferguson-Hessler, 1996; Jonassen, 1997). Thus, in PS-I contexts where no problem-solving support is offered, students who are confused by the problem parameters tend to focus on what seems familiar, which often leads to irrelevant solution paths (e.g., Kapur & Bielaczyc, 2012). Indeed, this deviant problem-solving pathway may be detrimental to learning as the activation of relevant prior knowledge would be more likely to encourage students to recognize their knowledge gaps than activating wide-range of, and often irrelevant, prior knowledge (Kapur, 2015).

It is thus important to design the features of PS-I to facilitate the representation of target concepts and the recognition of knowledge gaps in a knowledge element unit. In this regard, presenting contrasting cases or comparing students' solutions with canonical counterparts are well-established methods that instructors can apply. However, according to recent research by Loibl et al. (2020), merely engaging in the study of contrasting cases or the comparison solutions may not be sufficient to have students "engaging in the intended learning processes" (p. 133). More explicit and elaborate guidance might be necessary because there are many seemingly attractive elements with which students are obsessed or distracted in ill-structured problem-solving. For example, utilizing a knowledge graph where knowledge elements and their relationships are visualized can be an option that explicitly offers visual cues to students (Deng et al., 2022). However, such explicit visual representations of relevant knowledge may be insufficient for eliciting intended cognitive processes. More elaborate instructional support should be further investigated. For instance, in an unpublished dissertation (Lee, 2021), a visualized and personalized "Knowledge Gap Tracker", featuring self-assessed, visual cues for each knowledge element throughout PS-I phases, was suggested for facilitating the process of AKG at a knowledge element level. The Knowledge Gap Tracker enables students to actively self-assess their understanding of each knowledge element, reflect on their progress or areas that require attention. The Knowledge Gap Tracker then visualizes and highlights the specific areas and the extent to which students have knowledge gaps. This visual representation allows students to self-regulate their learning during asynchronous online instructions. The Knowledge Gap Tracker was shown to be effective in enhancing students' knowledge gap awareness and to effectively draw their attention to these areas of need.

Another important discussion with regard to cognitive and metacognitive processes is that students need evaluation criteria or explicit references to avoid making false judgements about their knowledge structures. For example, it was found that knowledge gaps were not recognized when students perceived their knowledge structure as *sufficient* when it was actually insufficient. That is, when reflection follows representation, there can still be cases where students do not perceive a gap between their own knowledge and the target knowledge. This is because the accuracy and extent of self-evaluation depend on whether students recognize their lack of ability or knowledge and the judgments they make during the problem-solving process, especially after the achievement of (sub-)goals. Some students may think they have sufficient knowledge to solve a given problem correctly even though their knowledge lacks essential components. This inaccurate "I Know" state is a matter of course in learning where no content-related instructional support is provided (Rawson & Dunlosky, 2007). Similarly, students' judgements of their knowledge structures as *naïve* also calls for a better understanding of AKG experiences. It was found that students were often uncertain whether their solutions were right or wrong. In this case, they have a global and naïve feeling-of-not-knowing which would be likely to end up not recognizing deep features unless proper guidance and instructions are accompanied or followed in the instruction phase.

The enhancement of students' abilities to self-assess their knowledge has been studied for several decades in educational research (Lai et al., 2018; Rawson & Dunlosky, 2007). Recently, equipping students with epistemic criteria has been emphasized as a crucial intervention that promotes productive knowledge development in inquiry-based learning (Pluta et al., 2011) and in strengthening the understanding of what constitutes a "good" argument and evidence (Duschl & Ellenbogen, 2009). Given the findings of this study, it is anticipated that many students would be able to avoid unproductive AKG experiences, and recognize their existing knowledge gaps if they are supported in establishing epistemic criteria throughout PS-I phases.

Limitations and suggestions for future research

The current study bears two main limitations. Firstly, the manifestations of AKG observed and analyzed in this study were limited to the problem-solving phase. As mentioned in our earlier caveats, AKG and subsequent deep feature recognition occurs throughout the two phases of PS-I. Thus, interpretation of our findings regarding the cognitive and metacognitive process of AKG experiences should be confined to the problem-solving phase. Secondly, it was conducted with a small number of participants in a physics learning session; thus, caution needs to be taken when generalizing its findings. Accordingly, we propose follow-up studies that explore two related avenues of inquiry. The first would expand the research scope from the problem-solving phase to the instruction phase to obtain a more comprehensible understanding of AKG experiences in PS-I contexts and thereby understand how different (meta-)cognitive process and corresponding AKG experiences may affect learning outcomes. The second would work with larger sample sizes to investigate the effects of instructional interventions following an AKG approach in various disciplines.

Data availability The datasets used during the current study are available from the corresponding author on reasonable request.

Declarations

Competing interests The authors declare that they have no competing interests.

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